1) $X \sim B(20, \frac{1}{2})$

Test whether coin is biased

$H_0 : p = \frac{1}{2}$

$H_1 : p <> \frac{1}{2}$

Two tailed test 5% each end

Expected value $= np = 10$

Actual number of heads $= 12$

$P(X \geq 12) = 1 - P(X \leq 11)$

$= 1 - 0.7483$

$= 0.2517$

$> 5\%$

$\therefore$ accept H_0 coin is fair

2) Suppose chicks male/female ratio $= 1$

$X \sim B(16, \frac{1}{2})$

where p = prob(female)

Expected value $X = 8$

Actual value $X = 13$

$H_0 : p = \frac{1}{2}$

$H_1 : p <> \frac{1}{2}$

Two tailed test $2\frac{1}{2}\%$ each end

Find $P(X \geq 13)$

$= 1 - P(X \leq 12)$

$= 1 - 0.9894$

$= 0.0106$

$< 2\frac{1}{2}\%$

Reject H_0

Accept H_1 male/female ratio $\neq 1$

3) Suppose people equally likely to choose left/right

$H_0 : p = \frac{1}{2}$ p = prob(left)

$H_1 : p <> \frac{1}{2}$

$X \sim B(12, \frac{1}{2})$

Expected value $np = 6$

Actual value $= 9$

Two tailed test $2\frac{1}{2}\%$ each end.

Find $P(X \geq 9)$

$= 1 - P(X \leq 8)$

$= 1 - 0.927$

$= 0.073$

$> 2\frac{1}{2}\%$

Accept H_0 equally likely to choose left or right

4) $H_0 : p = \frac{1}{4}$

$H_1 : p > \frac{1}{4}$

since people are complaining about rain

$X \sim B(20, \frac{1}{4})$

Do 1 tailed test at 5% level (say)

Expected value $= np = 5$

Actual value 10

Find $P(X \geq 10)$

$= 1 - P(X \leq 9)$

$= 1 - 0.9861$

$= 0.0139$

$< 5\%$

Reject H_0

Accept H_1 $p(\text{Rain}) > \frac{1}{4}$

5) $H_0 : p(\text{lemon}) = \frac{1}{6}$

$H_1 : p(\text{lemon}) <> \frac{1}{6}$

Two tailed test 5% each end.

$X \sim B(20, \frac{1}{6})$

Expected value $\frac{20}{6} = 3.33$

Actual value 6

Find $P(X \geq 6)$

$= 1 - P(X \leq 5)$

$= 1 - 0.8982$

$= 0.1018$

$> 5\%$

Accept H_0

6) $H_0 : p(\text{spade}) = \frac{1}{4}$

$H_1 : p(\text{spade}) < \frac{1}{4}$

$X \sim B(12, \frac{1}{4})$

One tailed test at 2% level

Expected value $12 \times \frac{1}{4} = 3$
Actual value $= 1$

Find $P(X \leq 1)$

$= 0.1584$

$> 2\%$

Accept H_0

7) $H_0 : p(\text{male}) = \frac{1}{4}$
$H_1 : p(\text{male}) <> \frac{1}{4}$

Two tailed test $2\frac{1}{2}\%$ each end.

$X \sim B(20, \frac{1}{4})$

Expected value $= 5$

$P(X=0) = 0.0032 \quad < 2\frac{1}{2}\%$
$P(X \leq 1) = 0.0243 \quad < 2\frac{1}{2}\%$
$P(X \leq 2) = 0.0913 \quad > 2\frac{1}{2}\%$

$P(X \geq 9) = 1 - P(X \leq 8)$
$= 1 - 0.9591$
$= 0.0409 \quad > 5\%$

$P(X \geq 10) = 1 - P(X \leq 9)$
$= 1 - 0.9861$
$= 0.0139 \quad < 5\%$

$X = 0, 1, 10, 11, 12, \ldots, 20$

would lead to rejecting H_0 and assuming different species

8) $H_0 : p(\text{question correct}) = \frac{1}{4}$
$H_1 : p <> \frac{1}{4}$
2 tail test 5% at each end

$X \sim B(20, \frac{1}{4})$

Expected value $= 20 \times \frac{1}{4} = 5$

$P(X=0) = 0.0032 \quad < 5\%$
$P(X \leq 1) = 0.0243 \quad < 5\%$
$P(X \leq 2) = 0.0913 \quad > 5\%$

$P(X \geq 9) = 1 - P(X \leq 8)$
$= 1 - 0.9591$
$= 0.0409 \quad < 5\%$

Reject H_0 for $X = 0, 1, 9, 10, 11, \ldots, 20$

9) H_0: p(letter a Z) = 0.15
H_0: p(Z) <> 0.15
Two tailed 5% at each end.
$X \sim B(50, 0.15)$

$P(X=0) = {}^{50}C_0\ 0.15^0\ 0.85^{50}$

$= 0.0003$

$P(X=1) = {}^{50}C_1\ 0.15^1\ 0.85^{49}$

$= 0.0026$

$P(X=2) = {}^{50}C_2\ 0.15^2\ 0.85^{48}$

$= 0.0113$

$P(X=3) = {}^{50}C_3\ 0.15^3\ 0.85^{47}$

$= 0.0319$

$P(X=4) = {}^{50}C_4\ 0.15^4\ 0.85^{46}$

$= 0.0661$

$P(X=5) = {}^{50}C_5\ 0.15^5\ 0.85^{45}$

$= 0.1072$

$P(X=6) = {}^{50}C_6\ 0.15^6\ 0.85^{44}$

$= 0.1419$

$P(X=7) = {}^{50}C_7\ 0.15^7\ 0.85^{43}$

$= 0.1575$

$P(X=8) = {}^{50}C_8\ 0.15^8\ 0.85^{42}$

$= 0.1493$

$P(X=9) = {}^{50}C_9\ 0.15^9\ 0.85^{41}$

$= 0.1230$

$P(X=10) = {}^{50}C_{10}\ 0.15^{10}\ 0.85^{40}$

$= 0.0890$

$P(X=11) = {}^{50}C_{11}\ 0.15^{11}\ 0.85^{39}$

$= 0.0571$

$P(X=12) = {}^{50}C_{12}\ 0.15^{12}\ 0.85^{38}$

$= 0.0328$

$P(X=13) = {}^{50}C_{13}\ 0.15^{13}\ 0.85^{37}$

$= 0.0169$

$P(X=14) = {}^{50}C_{14}\ 0.15^{14}\ 0.85^{36}$

$= 0.0079$

$P(X=15) = {}^{50}C_{15}\ 0.15^{15}\ 0.85^{35}$

$= 0.0033$

$P(X=16) = {}^{50}C_{16}\ 0.15^{16}\ 0.85^{34}$

$= 0.0013$

$P(X=0) = 0.0003$
$P(X \le 1) = 0.0029$
$P(X \le 2) = 0.0142$
$P(X \le 3) = 0.0461$
$P(X \le 4) = 0.1122$
$P(X \le 5) = 0.2194$
$P(X \le 6) = 0.3613$
$P(X \le 7) = 0.5188$
$P(X \le 8) = 0.6681$
$P(X \le 9) = 0.7911$
$P(X \le 10) = 0.8801$
$P(X \le 11) = 0.9372$
$P(X \le 12) = 0.9700$
$P(X \le 13) = 0.9869$
$P(X \le 14) = 0.9948$
$P(X \le 15) = 0.9981$

9 cont) $P(X \le 3) = 0.0461 < 5\%$

$P(X \geqslant 13) = 1 - P(X \le 12)$
$= 1 - 0.9700$
$= 0.03 < 5\%$

CRITICAL REGION

0, 1, 2, 3, 13, 14, 15, ..., 50

Reject H_0 for $X \le 3$ or $X \geqslant 13$

10) $X \sim B(25, 0.8)$

i) Expected value $25 \times 0.8 = 20$

ii) $P(X = 17) = {}^{25}C_{17}\, 0.8^{17}\, 0.2^{8}$

$= 0.0623$

iii) Find $P(Y \le 12)$

$P(X=0) = 0.0000$
$P(Y=1) = 0.0000$
$P(Y=2) = 0.0000$
$P(Y=3) = 0.0000$
$P(Y=4) = 0.0000$
$P(Y=5) = 0.0000$
$P(Y=6) = 0.0000$
$P(X=7) = 0.0000$
$P(X=8) = 0.0000$
$P(X=9) = 0.0000$
$P(X=10) = 0.0000$
$P(X=11) = 0.0001$
$P(Y=12) = 0.0003$

$P(X \le 12) = 0.0004$

Customer justified in complaint

11) $X \sim B(15, 0.5)$

i) $P(X=4) = {}^{15}C_4\, 0.5^4\, 0.5^{11}$
$= 0.0417$

ii) $P(X \le 4) = 0.0592$
from tables.

iii) $P(X=11) = {}^{15}C_{11}\, 0.5^{11}\, 0.5^4$

$= 0.0417$

$\therefore P(X = 4 \text{ or } 11) = 0.0834$

iv) $P(X \le 4) = 0.0592$
By Symmetry $P(X \geqslant 11) = 0.0592$

$P(X \le 4 \text{ or } X \geqslant 11) = 0.1184$

v) $H_0: p(\text{man}) = 0.5$
$H_1: p(\text{man}) <> 0.5$

2 tail test $2\frac{1}{2}\%$ at each end

Find $P(X \le 4) = 0.0592$
$> 2\frac{1}{2}\%$

Not sufficient evidence to reject H_0 so suppose process ok.

vi) $P(X \le 3) = 0.0176 < 2\frac{1}{2}\%$
$P(X \geqslant 12) = 0.0176 < 2\frac{1}{2}\%$

Judged acceptable for

$w = 4, 5, 6, 7, 8, 9, 10, 11$