MEI PURE 3 JANUARY 2003 VECTORS

4 The lines l_1 and l_2 have the following vector equations.

$$l_1: \quad \mathbf{r} = \begin{pmatrix} 1 \\ 8 \\ 8 \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ 3 \\ 4 \end{pmatrix}$$

$$l_2: \quad \mathbf{r} = \begin{pmatrix} 3 \\ 3 \\ -1 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}$$

(i) Verify that the point P (1, 2, 0) lies on both the lines l_1 and l_2. [3]

(ii) Find the acute angle between the two lines. [5]

A plane has cartesian equation $2x - y + z = 12$.

(iii) Write down a vector normal to the plane. Hence write down a vector equation for the line through P perpendicular to the plane. [2]

(iv) Find where this line meets the plane, and hence find the distance of P from the plane. [5]

[Total 15]

4 As part of a sculpture, an artist erects a flat triangular sheet ABC in his garden. The vertices are attached to vertical poles DA, EB and FC. The coordinate axes Ox and Oy are horizontal, and Oz is vertical. The coordinates of the triangle are A(2, 0, 2), B(−2, 0, 1) and C(0, 4, 3), with units in metres (see Fig. 4).

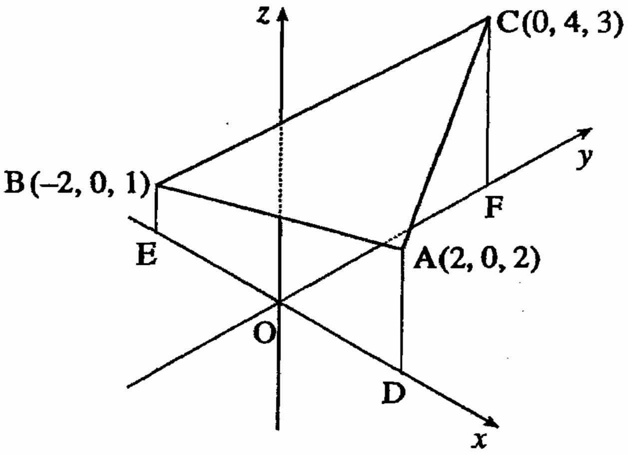

Fig. 4

(i) Find the length of the side AC. [2]

(ii) Find the scalar product $\overrightarrow{AB}.\overrightarrow{AC}$, and the angle BAC. [4]

(iii) Show that $\begin{pmatrix} 2 \\ 3 \\ -8 \end{pmatrix}$ is perpendicular to the lines AB and AC.

Hence find the cartesian equation of the plane ABC. [5]

(iv) The artist decides to erect another vertical pole GH based at the point G(1, 1, 0). Calculate the height of the pole if H is to lie in the plane ABC. [3]

[Total 14]

4 Fig. 4 shows a tetrahedron ABCD with vertices $A(1,0,0)$, $B(0,1,0)$, $C(0,0,2)$ and $D(-2,0,0)$.

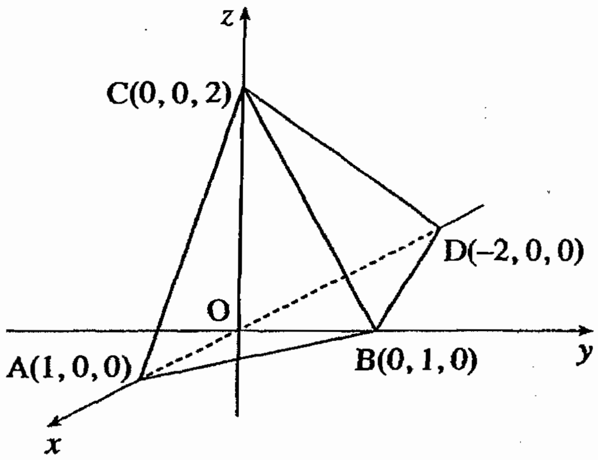

Fig. 4

(i) Find the scalar product $\overrightarrow{BA}.\overrightarrow{BC}$. Hence or otherwise find the angle ABC. [4]

(ii) Verify that the equation of the plane ABC is $2x + 2y + z = 2$. Write down a vector normal to the plane ABC. [4]

(iii) Write down a vector equation of the perpendicular l from the point D to the plane ABC. [2]

(iv) By finding where l meets the plane, find the distance from D to the plane ABC. [5]

[Total 15]

MEI PURE 3 JUNE 2004 VECTORS

3 The lines l_1 and l_2 have equations as follows:

$$l_1: \quad \mathbf{r} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix},$$

$$l_2: \quad \mathbf{r} = \begin{pmatrix} 0 \\ 2 \\ 2 \end{pmatrix} + \mu \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}.$$

(i) The point A lies on l_1 and has parameter $\lambda = 2$. The point B lies on l_2 and has parameter $\mu = 3$. Find the distance AB. [4]

(ii) Verify that the point $C(1, 1, 2)$ lies on l_1 and l_2.

Calculate angle ACB. [6]

(iii) Verify that the vector $\mathbf{n} = \begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix}$ is normal to the plane containing l_1 and l_2.

Hence or otherwise find the cartesian equation of this plane. [4]

[Total 14]

4 Fig. 4 shows the roof of a house. The coordinates of points A, B, C, D, E and F with respect to axes Ox, Oy and Oz are as shown in the diagram. ABCDE is a plane. All lengths are in metres.

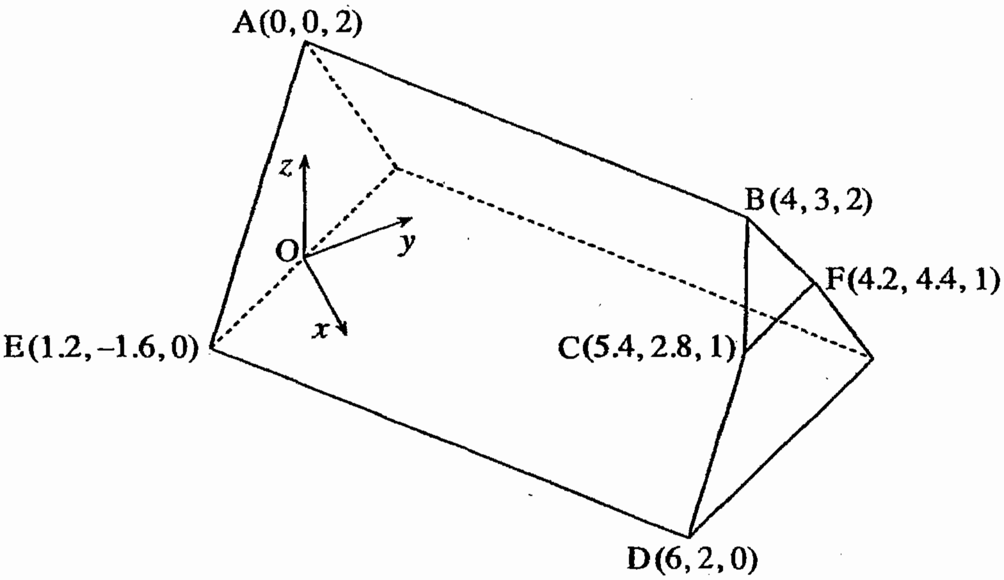

Fig. 4

(i) Write down the vectors $\overrightarrow{BC}$ and $\overrightarrow{BF}$. [1]

(ii) Show that triangle BCF is isosceles. [2]

(iii) Verify that the vector $\mathbf{n}_1 = \begin{pmatrix} 4 \\ 3 \\ 5 \end{pmatrix}$ is normal to the plane BCF. Deduce the equation of the plane BCF. [5]

(iv) Verify that the cartesian equation of the plane through A, B and D is

$$3x - 4y + 5z = 10.$$

Write down a vector $\mathbf{n}_2$ normal to this plane. [3]

(v) Find the angle between the planes BCF and ABCDE. [4]

4 Fig. 4 shows a triangular prism. The position vectors of A, B and C, relative to an origin O, are **a**, **b** and **c** respectively.

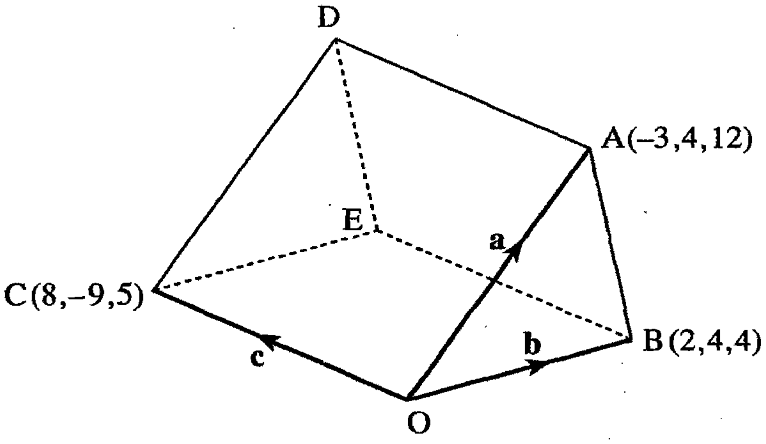

Fig. 4

(i) Write down the vectors **a** and **b**.

Calculate the angle AOB, and find the area of triangle AOB. [6]

The triangle CDE is the triangle OAB translated by the vector **c**.

(ii) Write down a vector equation of the line AD. [2]

(iii) Show that the vector **c** is perpendicular to the plane OAB. Hence find the cartesian equations of the planes OAB and CDE. [5]

(iv) Find the volume of the prism. [2]

[Total 15]