4 (i) $\begin{pmatrix}1\\8+3\lambda\\8+4\lambda\end{pmatrix}=\begin{pmatrix}1\\2\\0\end{pmatrix}$ when $\lambda=-2$	M1 A1	Equating $\lambda=-2$
$\begin{pmatrix}3+2\mu\\3+\mu\\-1-\mu\end{pmatrix}=\begin{pmatrix}1\\2\\0\end{pmatrix}$ when $\mu=-1$ So both lines contain the point (1, 2, 0)	A1 [3]	$\mu=-1$
(ii) Angle between $\begin{pmatrix}0\\3\\4\end{pmatrix}$ and $\begin{pmatrix}2\\1\\-1\end{pmatrix}$ $\text{Cos }\theta=\frac{0\times2+3\times1+4\times-1}{\sqrt{25\times6}}$ $=-\frac{1}{5\sqrt{6}}$ $\Rightarrow\theta=94.7°$ so $85.3°$	M1 M1 A1,A1 A1 [5]	Give M0 for other vectors but allow the second M1 and A1,A1 for $\cos\theta=$ $(1.3+8.3+8.-1)/\sqrt{(129.19)}$ -1 or 1, $5\sqrt{6}$ 85.3°, condone 94.7°. Accept 1.49 radians, condone 1.65 radians
(iii) $\mathbf{n}=\begin{pmatrix}2\\-1\\1\end{pmatrix}$	B1	
$\mathbf{r}=\begin{pmatrix}1\\2\\0\end{pmatrix}+\lambda\begin{pmatrix}2\\-1\\1\end{pmatrix}$	B1ft [2]	Condone the omission of **r**. ft their **n**
(iv)Substituting $x=1+2\lambda$, $y=2-\lambda$, $z=\lambda$; $\Rightarrow 2(1+2\lambda)-(2-\lambda)+\lambda=12$ $\Rightarrow 2+4\lambda-2+\lambda+\lambda=12$ $\Rightarrow\lambda=2$ So line meets plane at (5, 0, 2) Perp distance $=\sqrt{\{(5-1)^2+(0-2)^2+(2-0)^2\}}$ $=\sqrt{24}$ or $2\sqrt{6}$ or 4.90	M1 A1 ft A1 ft M1 A1c.a.o [5] Total [15]	 ft their line from (iii) or l_1 or l_2 $\lambda=2$ Distance formula.

4 (i) $AC = \sqrt{\{2^2 + (-4)^2 + (-1)^2\}}$ $= \sqrt{21}$ or 4.58…	M1 A1 [2]	
(ii) $\vec{AB} = \begin{pmatrix}-4\\0\\-1\end{pmatrix}$ $\vec{AC} = \begin{pmatrix}-2\\4\\1\end{pmatrix}$ $\vec{AB}.\vec{AC} = (-4)\times(-2) + 0\times 4 + (-1)\times 1$ $= 7$	M1 A1	For vectors **AB** and **AC**, (accept **BA**, **CA**. Condone one slip in each vector), and for evaluating the scalar product.soi 7 must be seen.
$\cos BAC = \dfrac{7}{\sqrt{17}\times\sqrt{21}} = 0.3704..$	M1	Ft their vectors and their scalar product.
$\Rightarrow BAC = 68.25°$	A1 [4]	Or 68.3° or 1.19 radians
(iii) $\begin{pmatrix}2\\3\\-8\end{pmatrix}.\begin{pmatrix}-4\\0\\-1\end{pmatrix} = -8+0+8=0$	E1	-8+8 must be seen
$\begin{pmatrix}2\\3\\-8\end{pmatrix}.\begin{pmatrix}-2\\4\\1\end{pmatrix} = -4+12-8=0$	E1	-4+12-8 must be seen
so perpendicular to AB and AC Equation of plane: $2x + 3y - 8z = c$ At A, $2\times 2 + 3\times 0 - 8\times 2 = c$ $\Rightarrow c = -12$ $\Rightarrow 2x + 3y - 8z = -12$	M1 DM1 A1 [5]	substituting the coordinates of A, B or C or using $\underline{a}.\underline{n}$ If a vector equation is used give M1 for a correct form, DM1 for eliminating the two parameters and A1 for the result
(iv) H is (1, 1, *h*) where $2\times 1 + 3\times 1 - 8\times h = -12$ $\Rightarrow 8h = 17$ $\Rightarrow h = 2.125$ (m)	M1 A1ft A1cao	Ft their equation from (iii)
<u>or</u> The equation of GH is $\begin{pmatrix}x\\y\\z\end{pmatrix} = \begin{pmatrix}1\\1\\0\end{pmatrix} + \lambda\begin{pmatrix}0\\0\\1\end{pmatrix}$	M1	
Meets AB where $2(1) + 3(1) - 8(\lambda) = -12$ $\lambda = 17/8$ and so h=2.125(m)	A1 ft A1 cao [3] Total 14	Ft their equation from (iii)

4(i) $\vec{BA} = \begin{pmatrix}1\\0\\0\end{pmatrix} - \begin{pmatrix}0\\1\\0\end{pmatrix} = \begin{pmatrix}1\\-1\\0\end{pmatrix}$ $\vec{BC} = \begin{pmatrix}0\\0\\2\end{pmatrix} - \begin{pmatrix}0\\1\\0\end{pmatrix} = \begin{pmatrix}0\\-1\\2\end{pmatrix}$ $\Rightarrow \vec{BA}.\vec{BC} = 1\times 0 + (-1)\times(-1) + 0\times 2$ $= 1$ $\cos ABC = \frac{\vec{BA}.\vec{BC}}{\lvert BA\rvert \lvert BC\rvert} = \frac{1}{\sqrt{2}.\sqrt{5}}$ $\Rightarrow ABC = 71.6°$	M1 A1 M1 A1[4]	Allow any two vectors Accept omission of working or correct scalar product seen in the formula for cos ABC
(ii) $2x + 2y + z = 2$ At A (1,0,0) $2\times 1 + 2\times 0 + 0 = 2$ at B (0,1,0) $2\times 0 + 2\times 1 + 0 = 2$ at C (0,0,2) $2\times 0 + 2\times 0 + 2 = 2$ $\Rightarrow$ Equation of the plane ABC is $2x + 2y + z = 2$ Normal vector is $\begin{pmatrix}2\\2\\1\end{pmatrix}$ Alternative schemes <u>Vector equation :</u> M1 for the correct form of a vector equation of the plane. M1 for the complete elimination of two parameters. A1 for deducing the correct equation of the plane B1 for the normal vector.	M1 B2,1.0 B1 [4]	Substituting the coordinates of one point into the eq'n. Accept eg 2+0+0 = 2 but not 2=2. $k\begin{pmatrix}2\\2\\1\end{pmatrix}$ Accept a row vector, but not the equation of a particular normal <u>The use of a vector product:</u> M1 A1 for the correct vector product of two vectors in the plane. B1 for identifying the normal A1 for the RHS of the equation <u>The use of scalar products:</u> B1 for the normal vector. M1 for the scalar product of the normal and one vector in the plane. A1 for two such products = 0 A1 for the RHS of the equation
(iii) $\mathbf{r} = \begin{pmatrix}-2\\0\\0\end{pmatrix} + \lambda\begin{pmatrix}2\\2\\1\end{pmatrix}$	B1 B1ft [2]	$\begin{pmatrix}-2\\0\\0\end{pmatrix} + \ldots$ $\ldots + \lambda\begin{pmatrix}2\\2\\1\end{pmatrix}$ Condone the omission of $\mathbf{r} =$
(iv) $\mathbf{r} = \begin{pmatrix}-2+2\lambda\\2\lambda\\\lambda\end{pmatrix}$: $2(-2+2\lambda) + 4\lambda + \lambda = 2$ $\Rightarrow 9\lambda = 6$ $\Rightarrow \lambda = 2/3$ So line meets plane at $(-\frac{2}{3}, \frac{4}{3}, \frac{2}{3})$. Distance = $\sqrt{(-2+\frac{2}{3})^2 + (-\frac{4}{3})^2 + (-\frac{2}{3})^2} = 2$	M1 A1 A1ft M1 A1 [5]	ft their equation in (iii) ft their λ and their equation Or $\lvert \frac{2\times(-2) + 2\times 0 + 1\times 0 - 2}{\sqrt{2^2+2^2+1^2}} \rvert = 2$

3 (i) A: $\begin{pmatrix}0\\1\\0\end{pmatrix}+2\begin{pmatrix}1\\0\\2\end{pmatrix}=\begin{pmatrix}2\\1\\4\end{pmatrix}$ so A is (2, 1, 4) B: $\begin{pmatrix}0\\2\\2\end{pmatrix}+3\begin{pmatrix}-1\\1\\0\end{pmatrix}=\begin{pmatrix}-3\\5\\2\end{pmatrix}$ so B is (−3, 5, 2) $AB^2=(2+3)^2+(1-5)^2+(4-2)^2$ $=45$ $\Rightarrow AB=\sqrt{45}=6.708\ldots$	B1 B1 M1 A1 cao [4]	Accept column vectors Distance formula. Ft their A and B. Condone one slip. Implied by 45 or the correct result √45 or 6.708…Accept 6.71
(ii) On l_1: $\begin{pmatrix}\lambda\\1\\2\lambda\end{pmatrix}=\begin{pmatrix}1\\1\\2\end{pmatrix}$ when $\lambda=1$.(by inspection) On l_2: $\begin{pmatrix}-\mu\\2+\mu\\2\end{pmatrix}=\begin{pmatrix}1\\1\\2\end{pmatrix}$ when $\mu=-1$. Angle ACB = angle between $\begin{pmatrix}1\\0\\2\end{pmatrix}$ and $\begin{pmatrix}-1\\1\\0\end{pmatrix}$ $\cos\theta=\dfrac{1\times(-1)+0\times1+2\times0}{\sqrt{5}\times\sqrt{2}}=-\dfrac{1}{\sqrt{10}}$ $\Rightarrow\ \theta=108.4°$	B1 B1 M1 M1 A1 A1 cao [6]	**Or from** $\begin{pmatrix}0\\1\\0\end{pmatrix}+\lambda\begin{pmatrix}1\\0\\2\end{pmatrix}=\begin{pmatrix}0\\2\\2\end{pmatrix}+\mu\begin{pmatrix}-1\\1\\0\end{pmatrix}$ $\Rightarrow \lambda=-\mu;\ 1=2+\mu;\ 2\lambda=2$ or use of **CA** = $\begin{pmatrix}1\\0\\2\end{pmatrix}$ and **CB** = $\begin{pmatrix}-4\\4\\0\end{pmatrix}$ ft their A and B Use of scalar product ft their vectors $-1/\sqrt{10}$ allow + for this A1but final answer must be obtuse .Accept 1.89 radians
(iii) $\begin{pmatrix}2\\2\\-1\end{pmatrix}.\begin{pmatrix}1\\0\\2\end{pmatrix}=2\times1+2\times0+(-1)\times2=0$ $\begin{pmatrix}2\\2\\-1\end{pmatrix}.\begin{pmatrix}-1\\1\\0\end{pmatrix}=2\times(-1)+2\times1+(-1)\times0=0$ so **n** is perpendicular to l_1 and l_2. Equation of plane is $2x+2y-z=d$ Substituting $x=1, y=1, z=2$, $\Rightarrow\ 2+2-2=d \Rightarrow d=2$ So plane is $2x+2y-z=2$.	B1 B1 M1 A1 [4] Total [14]	May also use **CB** = $\begin{pmatrix}-4\\4\\0\end{pmatrix}$ or **AB**= $\begin{pmatrix}-5\\4\\-2\end{pmatrix}$ or $\begin{pmatrix}0\\1\\0\end{pmatrix}-\begin{pmatrix}0\\2\\2\end{pmatrix}=\begin{pmatrix}0\\-1\\-2\end{pmatrix}$ (or use of the vector equation of the plane (in a correct form and elimination of the (parameters (or the correct evaluation of the vector (product of two appropriate vectors

4 (i) $\overrightarrow{BC}=\begin{pmatrix}1.4\\-0.2\\-1\end{pmatrix}$, $\overrightarrow{BF}=\begin{pmatrix}0.2\\1.4\\-1\end{pmatrix}$	B1 [1]	Accept row vectors
(ii) BC = $\sqrt{(1.4^2+0.2^2+1^2)}$ = BF so triangle BCF is isosceles	M1 A1w.w.w. [2]	Length formula applied to BC and BF SCB1 for <u>convincing</u> verbal argument
(iii) $\mathbf{n}_1.\overrightarrow{BC}=\begin{pmatrix}4\\3\\5\end{pmatrix}.\begin{pmatrix}1.4\\-0.2\\-1\end{pmatrix}=5.6-0.6-5=0$ $\mathbf{n}_1.\overrightarrow{BF}=\begin{pmatrix}4\\3\\5\end{pmatrix}.\begin{pmatrix}0.2\\1.4\\-1\end{pmatrix}=0.8+4.2-5=0$ Equation of plane is therefore $4x+3y+5z=d$ Substituting $x=4, y=3, z=2$: $\Rightarrow$ $d=16+9+10=35$ $\Rightarrow$ plane is $4x+3y+5z=35$.	M1 A1 A1 M1 A1 [5]	Scalar product with $\overrightarrow{BC}$ or with $CF=$ $\begin{pmatrix}4\\3\\5\end{pmatrix}.\begin{pmatrix}-1.2\\1.6\\0\end{pmatrix}=-4.8+4.8$ or with $\overrightarrow{BF}$ Allow one error if all three are done. OR M1 correct form of vector equation A1 correct vector equation M1 eliminate parameters A1 correct cartesian equation A1 confirm the normal
(iv) At A (0, 0, 2): 3×0 – 4×0 + 5×2 = 10 At B (4, 3, 2): 3×4 – 4×3 + 5×2 = 10 At D (6, 2, 0): 3×6 – 4×2 + 5×0 = 10 $\mathbf{n}_2=\begin{pmatrix}3\\-4\\5\end{pmatrix}$	M1 A1 B1 [3]	substituting one set of coordinates into equation all three done correctly OR M1 correct form of vector equation and elimination of parameters A1 correct Cartesian equation
(v) $\cos\theta=\dfrac{\mathbf{n}_1.\mathbf{n}_2}{\lvert\mathbf{n}_1\rvert.\lvert\mathbf{n}_2\rvert}$ $=\dfrac{\begin{pmatrix}4\\3\\5\end{pmatrix}.\begin{pmatrix}3\\-4\\5\end{pmatrix}}{\sqrt{50}.\sqrt{50}}=\dfrac{25}{50}=\dfrac{1}{2}$ $\Rightarrow$ $\theta=60°$.	M1 A1ft A1cao A1 [4]	Use of $\mathbf{n}_1$ and their $\mathbf{n}_2$ scalar product = 25 ft their $\mathbf{n}_2$ $\cos\theta=½$ allow 120°

4 (i) $\mathbf{a}=\begin{pmatrix}-3\\4\\12\end{pmatrix}$, $\mathbf{b}=\begin{pmatrix}2\\4\\4\end{pmatrix}$ $\Rightarrow \cos\theta=\frac{\mathbf{a.b}}{\lvert\mathbf{a}\rvert\lvert\mathbf{b}\rvert}=\frac{(-3)\times 2+4\times 4+12\times 4}{\sqrt{169}\times\sqrt{36}}$ $=\frac{58}{78}\Rightarrow\theta=41.96°$ (Accept 42° or 0.73 radians) Area of triangle OAB = $\frac{1}{2}\times 13\times 6\sin\theta$ = 26.1 (units ^{2}) (Accept 26)	M1 A1 A1 A1 M1 A1 ft [6]	use of scalar product (allow one slip) correct numerator correct denominator ft their θ
(ii) $\mathbf{r}=\begin{pmatrix}-3\\4\\12\end{pmatrix}+\lambda\begin{pmatrix}8\\-9\\5\end{pmatrix}$	B1 B1 [2]	$\mathbf{r}=\begin{pmatrix}-3\\4\\12\end{pmatrix}+\ldots$ or, using D, $\mathbf{r}=\begin{pmatrix}5\\-5\\17\end{pmatrix}+\ldots$ $\ldots+\lambda\begin{pmatrix}8\\-9\\5\end{pmatrix}$ o.e.
(iii) $\mathbf{c}.\overrightarrow{OA}=\begin{pmatrix}8\\-9\\5\end{pmatrix}.\begin{pmatrix}-3\\4\\12\end{pmatrix}=-24-36+60=0$ $\mathbf{c}.\overrightarrow{OB}=\begin{pmatrix}8\\-9\\5\end{pmatrix}.\begin{pmatrix}2\\4\\4\end{pmatrix}=16-36+20=0$ $\Rightarrow$ **c** is perpendicular to the plane OAB OAB: $8x-9y+5z=0$ CDE: $8x-9y+5z=d$ At C, $x=8, y=-9, z=5$ $\Rightarrow d=64+81+25=170$ $\Rightarrow 8x-9y+5z=170$	B1 B1 B1 M1 A1 [5]	Working must be seen
(iv) OC = √170 Volume of prism = 26.1 × √170 = 340 (units³)	M1 A1cao [2] [15]	