MEI MECHANICS 1 - REVISION

Projectiles

3 questions taken from the 1999 and 2000 examinations

together with worked solutions

2 A footballer is standing with the ball 12 m from an open goal, and the goalkeeper is lying on the ground 6 m from the goal line.

You may assume that air resistance is negligible and that the ball is at rest before it is kicked.

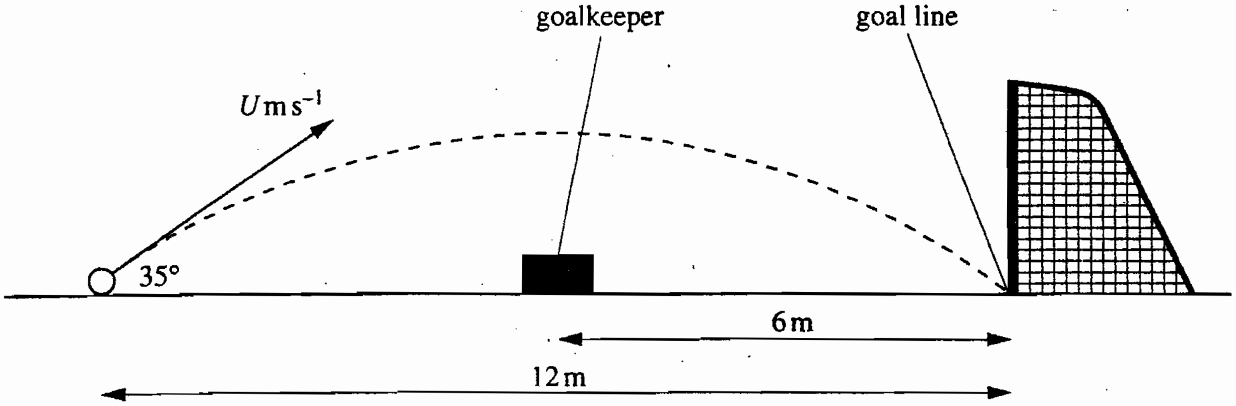

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Fig. 2.1

One strategy is to kick the ball so that it lands on the goal line, as shown in Fig. 2.1.

Suppose the footballer kicks the ball at U m s^{-1} at an angle of 35° to the horizontal so that it lands on the goal line after T seconds.

(i) Show that $UT \cos 35° = 12$. By considering the height of the ball, find another equation involving U and T. Show that T is about 1.31. [6]

The ball must be at least 1.6 m above the ground when it passes over the goalkeeper.

(ii) Verify that the ball will pass over the goalkeeper at an adequate height. [3]

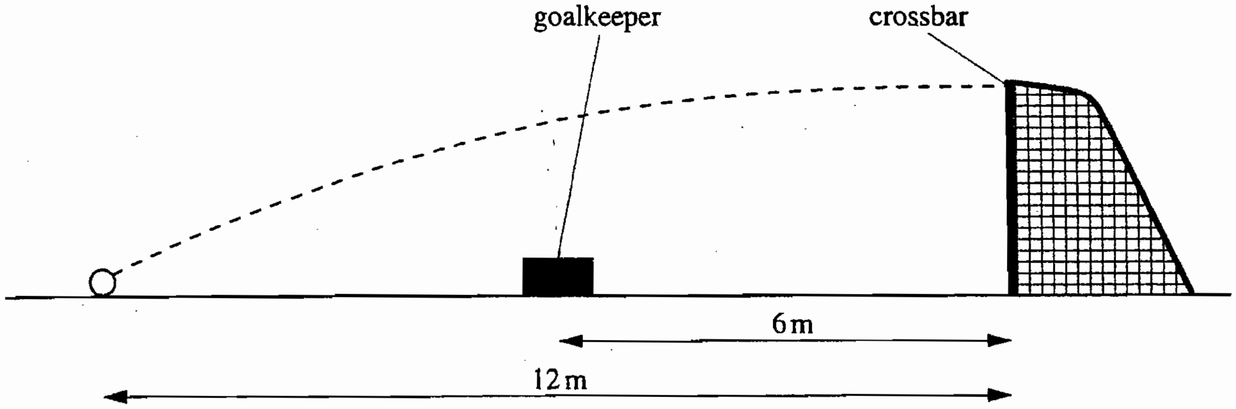

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Fig. 2.2

Another strategy is to kick the ball so that it passes just under the crossbar *at the top of its trajectory* having risen through a vertical height of 2.4 m, as shown in Fig. 2.2. [Note that the values of the speed and angle of projection are both different in this new situation.]

(iii) Determine how long it takes the ball to reach the goal and whether the ball passes over the goalkeeper at an adequate height. [6]

Total [15]

2 In this question take $g = 10 \text{ m s}^{-2}$ and neglect the effect of air resistance.

(i) A particle is projected vertically upwards and reaches a maximum height of 3.2 m above its point of projection. Show that the speed of projection of the particle is 8 m s^{-1}. Show also that, on the way down, the particle returns to the point of projection after 1.6 s. [5]

(ii) Explain briefly why a particle projected at *any* angle and reaching a maximum height of 3.2 m returns to the level from which it is projected after 1.6 s. [1]

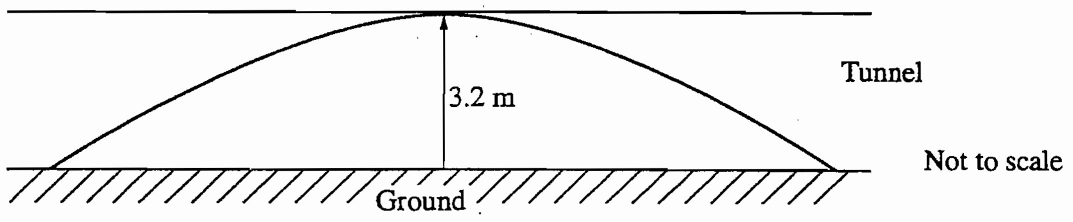

Fig. 2

A small stone is kicked from the floor of a horizontal tunnel of height 3.2 m with a speed of projection of 16 m s^{-1}. The stone just reaches the roof of the tunnel at its maximum height but does not touch it, as shown in Fig. 2.

(iii) Calculate the angle of projection and the horizontal range of the stone. [4]

(iv) Explain briefly why the range you have calculated is the maximum horizontal range for a stone kicked at the given speed in this tunnel. [1]

Suppose that the stone hits a vertical wall when it is 1 m above the floor and descending.

(v) Calculate the horizontal distance of the wall from the point of projection. [4]

[Total 15]

1

Two distress flares are fired from a life-raft at sea. They ignite as soon as they leave the raft and travel as projectiles. You may assume that air resistance is negligible. You should assume that the flares are fired from sea level.

The first flare is fired vertically upwards at 40 m s^{-1}.

(i) Calculate the greatest height reached by this flare. Calculate also the time taken to reach this height. [4]

A flare can be seen from the shore when it is at least 35 m above sea level.

(ii) Show that the first flare can be seen from the shore for about 6.2 s. [5]

The second flare is projected at V m s^{-1} at 60° to the horizontal.

(iii) Show that this flare takes about $0.088V$ seconds to reach its greatest height. [1]

This second flare can be seen from the shore for 4 seconds.

(iv) Show that V is approximately 37.8. [3]

(v) Calculate the horizontal distance travelled by this flare while it can be seen from the shore. [2]

NOTE: FLARES FIRED FROM SEA LEVEL

JANUARY 1999 Q2

JAN99Q2

i) Horizontally

$x = u_x \times t$

$x = U\cos 35\, t$

Given $x = 12$ when $t = T$

$\Rightarrow 12 = U\cos 35 \times T$

or $UT\cos 35^\circ = 12$ (1)

Vertically

$y = u_y t - \frac{1}{2}gt^2$

$y = U\sin 35^\circ \times t - 4.9t^2$

Given $y = 0$ when $t = T$

$\Rightarrow 0 = U\sin 35^\circ \times T - 4.9T^2$

From (1) $U = \dfrac{12}{T\cos 35}$ (2)

Substituting gives

$0 = \dfrac{12}{T\cos 35}\sin 35 T - 4.9T^2$

$0 = 12\tan 35 - 4.9T^2$

$4.9T^2 = 12\tan 35^\circ$

$T = \sqrt{\dfrac{12\tan 35}{4.9}}$

$T = 1.3095$

$T = 1.31$ s to 3 s.f.

ii) Ball above goalkeeper when $t = \dfrac{1.31}{2} = 0.655$ s

At this time

$y = U\sin 35 \times 0.655 - 4.9 \times 0.655^2$

From (2) $U = \dfrac{12}{1.31 \times \cos 35}$

$\therefore y = \dfrac{12}{1.31\cos 35} \times \sin 35 \times 0.655 - 4.9 \times 0.655^2$

$y = 6\tan 35^\circ - 4.9 \times 0.655^2$

$y = 2.099 = 2.10$ m to 3 s.f

$\therefore$ greater than required 1.6m

iii) Peak height $= 2.4$ m

Using $v^2 = u^2 + 2as$

$0^2 = u_y^2 - 19.6y$

$0^2 = u_y^2 - 19.6 \times 2.4$

$\Rightarrow u_y = \sqrt{19.6 \times 2.4} = 6.8586\ \text{ms}^{-1}$

Using $v = u + at$ at peak

$0 = u_y - 9.8t$

$t = \dfrac{u_y}{9.8} = \dfrac{6.8586}{9.8} = 0.6999$ s

$t = 0.700$ s to 3 s.f.

Reaches goal after 0.700 s

Above keeper after 0.350 s

$y = 6.8586 \times .350 - 4.9 \times 0.350^2$

$= 1.80$ m

$\therefore$ high enough

JANUARY 2000 Q2

Jan00Q2

i) Take $g = 10\ ms^{-2}$

Using $v^2 = u^2 + 2as$

$$v_y^2 = u_y^2 - 20y$$

At peak $v_y = 0$

$$0 = u_y^2 - 20 \times 3.2$$

$$u_y^2 = 64$$

$$\Rightarrow u_y = 8\ ms^{-1}$$

Vertically using $s = ut + \frac{1}{2}at^2$

$$y = 8t - 5t^2$$

On landing $y = 0$

$$0 = 8t - 5t^2$$

$$0 = t(8 - 5t)$$

$$\Rightarrow t = 0 \quad \text{or} \quad 8 - 5t = 0$$

$$t = \frac{8}{5}$$

$$t = 1.6\ s$$

Lands when $t = 1.6\ s$

ii) If particle reaches max. height of 3.2 m it must have had the same vertical component of initial velocity. Motion in the vertical direction will therefore be the same, including the time of flight

iii) $u_y = u \sin\theta$

But from part (i) $u_y = 8\ ms^{-1}$

$$\therefore 8 = 16 \sin\theta$$

$$\Rightarrow \sin\theta = \tfrac{1}{2}, \quad \underline{\theta = 30^\circ}$$

Horizontally,

$$x = u\cos\theta \times t$$

from part i $t = 1.6\ s$ for flight

$$x = 16 \cos 30 \times 1.6$$

$$x = 22.17\ m$$

Range $= 22.2\ m$ to 3 s.f.

iv) Used all available height. Max range would come from 45° launch but if angle was increased beyond 30° stone would hit roof

v)

$$y = 8t - 5t^2$$

When $y = 1$

$$1 = 8t - 5t^2$$

$$5t^2 - 8t + 1 = 0$$

By calc $t = 0.1336$ or $t = 1.4633$

Descending so $t = 1.4633$

$$x = u\cos\theta \times t$$

$$x = 16 \cos 30 \times 1.4633$$

$$x = 20.276\ m$$

Horiz distance $= 20.3\ m$ to 3 s.f.

JUNE 2000 Q2

JUN00Q2

i) Using $v^2 = u^2 + 2as$

At peak $0^2 = u_y^2 - 19.6y$

$19.6y = u_y^2 = 40^2$

$y = \dfrac{40^2}{19.6} = 81.633$ m

$y = 81.6$ m to 3 s.f.

Max height $= 81.6$ m

Using $v = u + at$

At max height $0 = 40 - 9.8t$

$t = \dfrac{40}{9.8} = 4.08$ s

Time to max height $= 4.08$ s

ii) Using $s = ut + \frac{1}{2}at^2$

$y = 40t - 4.9t^2$

when $y = 35$

$35 = 40t - 4.9t^2$

$4.9t^2 - 40t + 35 = 0$

By calc $t = 7.1665$ s
or $t = 0.9966$ s

Visible between these times

for $7.1665 - 0.9966 = 6.1699$ s

Visible for approx 6.2 s

iii) Vertically $v_y = u_y - 9.8t$

At peak $v_y = 0$

$0 = V\sin 60 - 9.8t$

$\Rightarrow t = \dfrac{V\sin 60}{9.8} = 0.088V$ s

iv)

$y = V\sin 60\, t - 4.9t^2$

At height 35m

$35 = \frac{\sqrt{3}}{2}Vt - 4.9t^2$

$4.9t^2 - \frac{\sqrt{3}}{2}Vt + 35 = 0$

$t = \dfrac{\frac{\sqrt{3}}{2}V \pm \sqrt{\frac{3}{4}V^2 - 4\times 4.9\times 35}}{9.8}$

$t = \dfrac{\frac{\sqrt{3}}{2}V}{9.8} \pm \dfrac{\sqrt{\frac{3}{4}V^2 - 686}}{9.8}$

Difference between roots $= 4$ s

$\Rightarrow \dfrac{\sqrt{\frac{3}{4}V^2 - 686}}{9.8} = 2$

$\Rightarrow \sqrt{\frac{3}{4}V^2 - 686} = 19.6$

$\frac{3}{4}V^2 - 686 = 19.6^2$

$\frac{3}{4}V^2 = 19.6^2 + 686$

$V = \sqrt{\frac{4}{3}(19.6^2 + 686)}$

$V = 37.774 \approx 37.8$ m s^{-1}

v)

horiz dist $= V\cos 60 \times$ time interval

$= 37.8 \times \cos 60^\circ \times 4$

$= 75.6$ m