6.5 FURTHER CONCEPTS FOR ADVANCED MATHEMATICS, FP1 (4755) AS

Objectives

To develop an understanding of the rigour and technical accuracy needed for more advanced study of mathematics.

Assessment

Examination: (72 marks)
1 hour 30 minutes.
The examination paper has two sections.

Section A: 5-7 questions, each worth at most 8 marks.
Section Total: 36 marks.

Section B: three questions, each worth about 12 marks.
Section Total: 36 marks.

Assumed Knowledge

Candidates are expected to know the content for *C1* and *C2*.

Subject Criteria

This unit is required for Advanced Subsidiary Further Mathematics. Candidates proceeding to Advanced GCE Further Mathematics will also need *FP2*.

The Units *C1, C2, C3* and *C4* are required for Advanced GCE Mathematics in order to ensure coverage of the subject criteria.

Calculators

In the MEI Structured Mathematics specification, no calculator is allowed in the examination for *C1*. For all other units, including this one, a graphical calculator is allowed.

FURTHER CONCEPTS FOR ADVANCED MATHEMATICS, FP1		
Specification	**Ref.**	**Competence Statements**
COMPLEX NUMBERS		
Quadratic equations.	FP1j1	Be able to solve any quadratic equation with real coefficients.
Addition, subtraction, multiplication and division of complex numbers.	2	Understand the language of complex numbers.
	3	Be able to add, subtract, multiply and divide complex numbers given in the form: $x + y\mathrm{j}$, where x and y are real.
	4	Know that a complex number is zero if and only if both the real and imaginary parts are zero.
Application of complex numbers to the solution of polynomial equations with real coefficients.	5	Know that the complex roots of real polynomial equations with real coefficients occur in conjugate pairs.
	6	Be able to solve equations of higher degree with real coefficients in simple cases.
	7	Know how to represent complex numbers and their conjugates on an Argand diagram.
	8	Be able to represent the sum and difference of two complex numbers on an Argand diagram.
Modulus-argument form	9	Be able to represent a complex number in modulus-argument form.
Simple loci in the Argand diagram.	10	Be able to represent simple sets of complex numbers as loci in the Argand diagram.
CURVE SKETCHING		
Treatment and sketching of graphs of rational functions.	FP1C1	Be able to sketch the graph of $y = \mathrm{f}(x)$ obtaining information about symmetry, asymptotes parallel to the axes, intercepts with the co-ordinate axes, behaviour near $x = 0$ and for numerically large x.
	2	Be able to ascertain the direction from which a curve approaches an asymptote.
	3	Be able to use a curve to solve an inequality.

FURTHER CONCEPTS FOR ADVANCED MATHEMATICS, FP1			
Ref.	**Notes**	**Notation**	**Exclusions**
	COMPLEX NUMBERS		
FP1j1		$\mathrm{j}^2 = -1$	
2	Real part, imaginary part, complex conjugate, modulus, argument, real axis, imaginary axis.	$z = x + y\mathrm{j}$ $z^* = x - y\mathrm{j}$ $\mathrm{Re}(z) = x$ $\mathrm{Im}(z) = y$	
3	Division using complex conjugates.		
4			
5	e.g. to solve a cubic equation given one complex root.		Equations with degree > 4. Equations with more than 2 complex roots (unless they are purely imaginary).
6			
7			
8			
9	Conversion between the forms $z = x + y\mathrm{j}$ and $z = r(\cos\theta + \sin\theta\,\mathrm{j})$.	Modulus of z: $\lvert z\rvert$ Radian measure.	
10	Circles of the form $\lvert z - a\rvert = r$. Half lines of the form $\arg(z - a) = \theta$.		
	CURVE SKETCHING		
FP1C1	Cases where $\mathrm{f}(x) = \frac{\mathrm{g}(x)}{\mathrm{h}(x)}$ and the order of $\mathrm{g}(x) \leq$ the order of $\mathrm{h}(x)$.		Oblique asymptotes.
2	Vertical and horizontal asymptotes.		Formal treatment.
3			

FURTHER CONCEPTS FOR ADVANCED MATHEMATICS, FP1		
Specification	**Ref.**	**Competence Statements**
PROOF		
Meaning of the terms if, only if, necessary and sufficient.	FP1p1	Be able to use the terms *if*, *only if*, *necessary* and *sufficient* correctly in any appropriate context.
Identities.	2	Know the difference between an equation and an identity.
	3	Be able to find unknown constants in an identity.
Proof by induction.	4	Be able to construct and present a correct proof using mathematical induction.
ALGEBRA		
Summation of simple finite series.	FP1a1	Know the difference between a sequence and a series.
	2	Be able to sum a simple series.
	3	Know the meaning of the word *converge* when applied to either a sequence or a series.
The manipulation of simple algebraic inequalities.	4	Be able to manipulate simple algebraic inequalities, to deduce the solution of such an inequality.
Relations between the roots and coefficients of quadratic, cubic and quartic equations.	5	Appreciate the relationship between the roots and coefficients of quadratic, cubic and quartic equations.
	6	Be able to form a new equation whose roots are related to the roots of a given equation by a linear transformation.

FURTHER CONCEPTS FOR ADVANCED MATHEMATICS, FP1			
Ref.	**Notes**	**Notation**	**Exclusions**
	PROOF		
FP1p1			
2	An identity is true for all values of the variables, an equation for only certain values.	$\equiv$	
3	By substituting particular values for the variable.		
4	Proofs of the sums of simple series. The result to be proved will always be given explicitly.		Proofs of divisibility.
	ALGEBRA		
FP1a1			
2	Using standard formulae for Σr, Σr^2 and Σr^3. Using the method of differences.		Derivation of partial fractions.
3			
4	Including those expressible in the form $f(x) > 0$ where $f(x)$ can be expressed as a product of linear factors.		Inequalities involving algebraic fractions where the numerator and denominator are of order ≥ 2.
5		Roots $\alpha, \beta, \gamma, \delta$.	Equations of degree ≥ 5.
6			New equations with non-linear combinations of roots.

FURTHER CONCEPTS FOR ADVANCED MATHEMATICS, FP1		
Specification	**Ref.**	**Competence Statements**
MATRICES		
Matrix addition and multiplication.	FP1m1	Be able to add, subtract and multiply conformable matrices, and to multiply a matrix by a scalar.
	2	Know the zero and identity matrices, and what is meant by equal matrices.
	3	Know that matrix multiplication is associative but not commutative.
Linear transformations in a plane and their associated 2x2 matrices.	4	Be able to find the matrix associated with a linear transformation and vice-versa.
Combined transformations in a plane.	5	Understand successive transformations and the connection with matrix multiplication.
Invariance.	6	Understand the meaning of invariant points and lines of invariant points in a plane and how to find them.
Determinant of a matrix.	7	Be able to find the determinant of a 2x2 matrix.
	8	Know that the determinant gives the area scale factor of the transformation, and understand the significance of a zero determinant.
The meaning of the inverse of a square matrix.	9	Understand what is meant by an inverse matrix.
	10	Be able to find the inverse of a non-singular 2x2 matrix.
The product rule for inverses.	11	Appreciate the product rule for inverse matrices.
Solution of equations.	12	Know how to use matrices to solve linear equations.
	13	In the case of 2 linear equations in 2 unknowns, be able to give a geometrical interpretation of a case where the matrix is singular.

FURTHER CONCEPTS FOR ADVANCED MATHEMATICS, FP1			
Ref.	**Notes**	**Notation**	**Exclusions**
	MATRICES		
FP1m1	Matrices of any suitable order.	$\mathbf{M} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$	
2		**O** (zero) **I** (identity).	
3			
4	Reflection in the x and y axes and in the lines $y = \pm x$. Rotation centre the origin through an angle θ. Enlargement centre the origin.	Column vectors for the position of points.	
5			
6			
7		$\begin{vmatrix} a & c \\ b & d \end{vmatrix}$ or det **M**.	
8	The terms *singular* and *non-singular*.		
9	Square matrices of any order.	$\mathbf{M}^{-1}$ (inverse).	
10	Candidates should know that: $\mathbf{AB} = k\mathbf{I} \Rightarrow \mathbf{A}^{-1} = \frac{1}{k}\mathbf{B}$.	The inverse of **M** is $\frac{1}{\det \mathbf{M}} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$	
11	$(\mathbf{AB})^{-1} = \mathbf{B}^{-1}\mathbf{A}^{-1}$.		
12			Equations with 3 or more unknowns when the inverse matrix is not known.
13	The graphs of the equations are parallel lines.		