Oxford Cambridge and RSA Examinations

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS

FURTHER CONCEPTS FOR ADVANCED MATHEMATICS, FP1 **4755**

Specimen Paper

Additional materials: Answer booklet
Graph paper
MEI Examination Formulae and Tables (MF 2)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, Centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- You **may** use a graphical or scientific calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The total number of marks for this paper is **72**.

Section A (36 marks)

1 Find the values of A, B and C in the identity $x^2 = A(x-1)^2 + B(x-2) + C$. **[3]**

2 Solve the inequality $x^2 \geq \frac{1}{x}$. **[4]**

3 Matrices **A** and **B** are given by:

$$\mathbf{A} = \begin{pmatrix} 1 & 0 & k \\ 0 & 2 & 1 \\ 0 & 0 & 3 \end{pmatrix} \text{ and } \mathbf{B} = \begin{pmatrix} 6 & 0 & -2k \\ 0 & 3 & -1 \\ 0 & 0 & 2 \end{pmatrix}.$$

Find the matrix product **AB**.
Hence write down the inverse of matrix **A** in the case when $k = 3$. **[4]**

4 A complex number α is given by $\alpha = 1 + 5\text{j}$.

(i) Find the modulus of α. **[1]**

(ii) Write down the complex conjugate α^*. **[1]**

(iii) Write down the value of $\alpha\alpha^*$. **[1]**

(iv) Express $\frac{\alpha + \alpha^*}{\alpha^*}$ in the form $a + b\text{j}$. **[2]**

5 The matrix $\mathbf{A} = \begin{pmatrix} -\frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} \end{pmatrix}$ defines a transformation in the (x, y)-plane.

(i) Find $\mathbf{A}^2$ and $\mathbf{A}^3$. **[3]**

(ii) Describe fully the transformation represented by **A**. **[3]**

6 Find $\sum_{r=1}^{n} r(6r+1)$, giving your answer in a fully factorised form. **[6]**

7 The quadratic equation $x^4 + px^3 + qx^2 + rx + s = 0$ has roots α, $-\alpha$, β and β.

(i) Express p, q, r and s in terms of α and β, simplifying your answers. **[6]**

(ii) Hence show that $pr - 4s = 0$. **[2]**

Section B (36 marks)

8 A curve has equation $y = \dfrac{x(x-1)}{(x+2)(x-3)}$.

(i) Write down the values of x for which $y = 0$. **[1]**

(ii) Write down the equations of the 3 asymptotes. **[3]**

(iii) Describe the behaviour of the curve for large positive and large negative values of x, justifying your description. **[3]**

(iv) Sketch the curve. **[3]**

(v) The equation $\dfrac{x(x-1)}{(x+2)(x-3)} = k$ has no real roots.
What can you say about the value of k? **[4]**

9 **(i)** Given that $\alpha = -1 + 2\mathrm{j}$, express α^2 and α^3 in the form $a + b\mathrm{j}$.
Hence show that α is a root of the cubic equation: $z^3 + 7z^2 + 15z + 25 = 0$. **[5]**

(ii) Find the other two roots of this cubic equation. **[4]**

(iii) Illustrate the three roots of the cubic equation on an Argand diagram. **[2]**

10 Prove by induction that $\sum_{r=1}^{n}(3r^2 - r) = n^2(n+1)$ for all positive integers, n. **[11]**