MEI

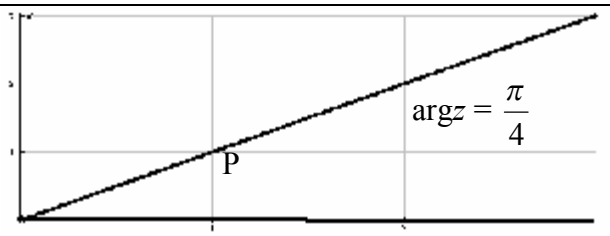
Mathematics in Education and Industry

MEI STRUCTURED MATHEMATICS

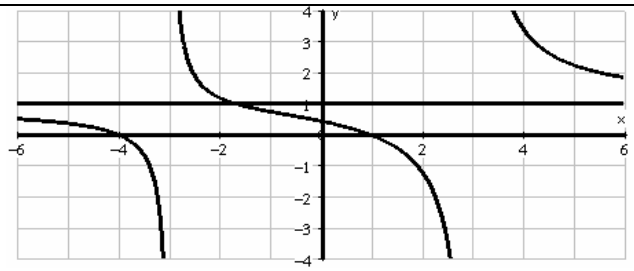
FURTHER CONCEPTS FOR ADVANCED MATHEMATICS, FP1

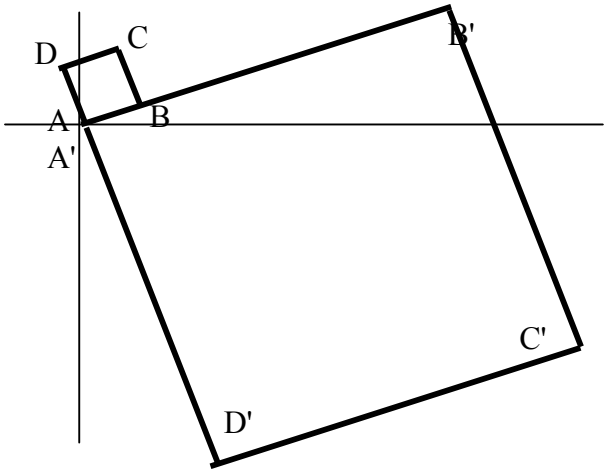
Practice Paper FP1-D

MARK SCHEME

Qu		Answer	Mark	Comment
Section A				
1		$\frac{5\text{j}}{(3-\text{j})}.\frac{(3+\text{j})}{(3+\text{j})}$ $=\frac{15\text{j}-5}{10}=-\frac{1}{2}+\frac{3}{2}\text{j}$	M1 A1 A1 **3**	Mult top and bottom Correct conjugate c.a.o
2		$\mathbf{M}=\begin{pmatrix}2 & 3\\ -1 & -4\end{pmatrix}\Rightarrow\mathbf{M}^{-1}=-\frac{1}{5}\begin{pmatrix}-4 & -3\\ 1 & 2\end{pmatrix}$ $=\begin{pmatrix}0.8 & 0.6\\ -0.2 & -0.4\end{pmatrix}$	M1 A1 A1 A1 **4**	Determinant Matrix
3	**(i)**	$\sum\alpha=0,$ $\sum\alpha\beta=1$	B1 B1 **2**	
	(ii)	$\sum\alpha^2=(\alpha+\beta+\gamma+\delta)^2=\left(\sum\alpha\right)^2-2\sum\alpha\beta=-2$	M1 A1 **2**	
4	**(i)**		B1 B1 **2**	Any line Correct angle
	(ii)	$1+\text{j}=\sqrt{2}\left(\cos\frac{\pi}{4}+\text{j}\sin\frac{\pi}{4}\right)$	M1 A1 **2**	
	(iii)	arg(1+j) = $\frac{\pi}{4}$ i.e. on line	B1 **1**	
5		$\frac{(x-3)(x+2)}{x+1}>0\Rightarrow(x-3)(x+2)>0$ if $x+1>0$ $\Rightarrow x>3$ or $(x-3)(x+2)<0$ if $x+1<0$ $\Rightarrow -2<x<-1$	M1 A1 A1 M1 A1 A1 **6**	Alternatively, multiply by $(x+1)^2$
6	**(i)**	$z_2=2-\text{j}$	B1 **1**	
	(ii)	$\frac{1}{z_1}+\frac{1}{z_2}=\frac{1}{2+\text{j}}+\frac{1}{2-\text{j}}=\frac{2-\text{j}+2+\text{j}}{4-\text{j}^2}$ $=\frac{4}{5}$	M1 A1 A1 **3**	 c.a.o.

	(iii)	$z_1^2 = 4+4\mathrm{j}+\mathrm{j}^2 = 3+4\mathrm{j}$ $z_2^2 = 4-4\mathrm{j}+\mathrm{j}^2 = 3-4\mathrm{j}$ $z_1^2 + z_2^2 = 6 \Rightarrow \mathrm{Im}\left(z_1^2 + z_2^2\right) = 0$ $z_1^2 - z_2^2 = 8\mathrm{j} \Rightarrow \mathrm{Re}\left(z_1^2 - z_2^2\right) = 0$	B1(both) B1(both) **2**	
7		Assume that the formula is true for $n = k$ i.e. $\sum_1^k r^3 = \frac{1}{4}k^2(k+1)^2$ Then $\sum_1^{k+1} r^3 = \frac{1}{4}k^2(k+1)^2 + (k+1)^3$ $= \frac{1}{4}(k+1)^2(k^2+4k+4)$ $= \frac{1}{4}(k+1)^2(k+2)^3$ So if true for $n = k$ then true also for $n = k+1$ But it is true for $n = 1$ since $1^3 = \frac{1}{4}1^2 2^2$ So true for all positive values of n.	B1 M1 A1 A1 B1 M1 A1 A1 **8**	
Section B				
8	**(i)**	$\frac{1}{5r-2} - \frac{1}{5r+3} = \frac{5r+3-(5r-2)}{(5r-2)(5r+3)}$ $= \frac{5}{(5r-2)(5r+3)}$	M1 A1 **2**	
	(ii)	$\sum_1^n \left(\frac{1}{(5r-2)(5r+3)}\right) = \frac{1}{5}\left(\sum\left[\frac{1}{5r-2} - \frac{1}{5r+3}\right]\right)$ $= \frac{1}{5}\left(\frac{1}{3} - \frac{1}{8}\right)$ $+ \frac{1}{5}\left(\frac{1}{8} - \frac{1}{13}\right)$ $+ \ldots\ldots\ldots\ldots$ $+ \frac{1}{5}\left(\frac{1}{5n-2} - \frac{1}{5n+3}\right)$ $= \frac{1}{5}\left(\frac{1}{8} - \frac{1}{5n+3}\right) = \frac{n}{3(5n+3)}$	M1 A1 M1 A1 A1 A1 A1 **7**	Use of (i) Deal with 5 Do the sum by writing out the terms

	(iii)	For $n = 2$, $\sum_{1}^{2}\left(\frac{1}{(5r-2)(5r+3)}\right) = \frac{1}{5}\left(\frac{1}{3}-\frac{1}{8}\right) + \frac{1}{5}\left(\frac{1}{8}-\frac{1}{13}\right)$ $= \frac{1}{5}\left(\frac{1}{3}-\frac{1}{13}\right) = \frac{2}{39}$ and $\frac{n}{3(5n+3)} = \frac{2}{3\times 13} = \frac{2}{39}$	M1 E1 **2**	Both
9	**(i)**	f(x) = 0 when $x^2 + 3x - 4 = 0$. i.e. x = -4, 1	M1 A1 **2**	
	(ii)	x = 3 and x = -3	B1 B1 **2**	
	(iii)	$1 + \frac{3x+5}{x^2-9} \equiv \frac{x^2-9+3x+5}{x^2-9} \equiv \frac{x^2+3x-4}{x^2-9}$ Horizontal asymptote is y = 1	M1 E1 B1 **3**	
	(iv)		B1 B1 B1 **3**	B1 for each branch
	(v)	From the graph f(x) ≥ 1 when $x > 3$ and $-3 < x < p$ where p is to be found. $\frac{x^2+3x-4}{x^2-9} = 1$ when 3x+5 = 0 $\Rightarrow x = -\frac{5}{3}$ $\left(\text{i.e.} p = -\frac{5}{3}\right)$ i.e. $-3 < x \leq -\frac{5}{3}$	M1 A1 A1 **3**	

10	**(i)**	$\begin{matrix} & A & B & C & D & & A' & B' & C' & D' \end{matrix}$ $\begin{pmatrix} 3 & 4 \\ 4 & -3 \end{pmatrix}\begin{pmatrix} 0 & 2 & 1 & -1 \\ 0 & 1 & 3 & 2 \end{pmatrix} = \begin{pmatrix} 0 & 10 & 15 & 5 \\ 0 & 5 & -5 & -10 \end{pmatrix}$ A' (0,0), B' (10.5) C' (15, -5) D' (5, -10)	M1 A3,2,1 **4**	-1 each error
	(ii)		B1 B1 **2**	Original image
	(iii)	Det **M** = 3×-3 - 4×4 = -25 Area ABCD = 1, Area A'B'C'D' = 25 $25 = \lvert -25 \rvert \times 1$	B1 E1 **2**	
	(iv)	$P = \begin{pmatrix} 5 & 0 \\ 0 & 5 \end{pmatrix}$	B1 **1**	
	(v)	P is enlargement, centre (0, 0), scale factor 5 Q is reflection in line $y = \frac{1}{2}x$	B1 B1 B1 **3**	All Reflection Equation of line (accept AB)