MEI

Mathematics in Education and Industry

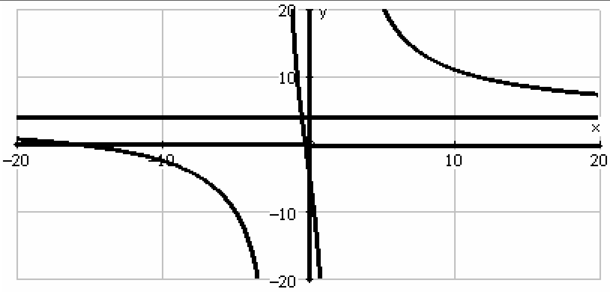
MEI STRUCTURED MATHEMATICS

FURTHER CONCEPTS FOR ADVANCED MATHEMATICS, FP1

Practice Paper FP1-C

MARK SCHEME

Qu		Answer	Mark	Comment
Section A				
1		$\sum_1^n (r+1)(r+2) = \sum_1^n (r^2+3r+2)$ $= \frac{n}{6}(n+1)(2n+1) + 3\frac{n}{2}(n+1) + 2n$ $= \frac{n}{6}(2n^2+12n+22) = \frac{n}{3}(n^2+6n+11)$	M1 A1 A1 A1 A1 **5**	
2		$(x+2) < \frac{4x}{(x-3)}$ If $x>3$, $(x+2)(x-3) < 4x \Rightarrow x^2-5x-6<0$ $\Rightarrow (x-6)(x+1)<0$ $\Rightarrow 3<x<6$ If $x<3$, $(x-6)(x+1)>0$ $\Rightarrow x<-1$	M1 A1 A1 A1 A1 **5**	Or × $(x-3)^2$ Then deal with cubic Or graphically
3	**(i)**	$x^3+2x^2+9x+18=0$	B1 **1**	
	(ii)	$f(x)=x^3+2x^2+9x+18$; $f(-2)=0$ $\Rightarrow x=-2$ is a root.	B1 **1**	
	(iii)	$x^3+2x^2+9x+18=0 \Rightarrow (x+2)(x^2+9)=0$ $\Rightarrow$ The other two roots are $3\mathrm{j}$ and $-3\mathrm{j}$.	M1 A1 A1 **3**	
4		$(a+b\mathrm{j})(a-b\mathrm{j})-2\mathrm{j}(a+b\mathrm{j})=7-4\mathrm{j}$ $\Rightarrow a^2+b^2-2\mathrm{j}a+2b=7-4\mathrm{j}$ $\Rightarrow (a^2+b^2+2b)-2\mathrm{j}a=7-4\mathrm{j}$ $\Rightarrow (a^2+b^2+2b)=7$ and $a=2$ $\Rightarrow b^2+2b-3=0 \Rightarrow (b+3)(b-1)=0$ $\Rightarrow b=1$ or -3 $\Rightarrow z=2+\mathrm{j}$ or $z=2-3\mathrm{j}$	M1 A1 A1 A1 (for *a*) M1 A1(both) **6**	
5		$\alpha+\beta+\gamma=9$, $\alpha\beta+\beta\gamma+\gamma\alpha=3$, $\alpha\beta\gamma=39$ $(\alpha-3)+(\beta-3)+(\gamma-3)=\alpha+\beta+\gamma-9=0$ $(\alpha-3)(\beta-3)+(\beta-3)(\gamma-3)+(\gamma-3)(\alpha-3)$ $=(\alpha\beta+\beta\gamma+\gamma\alpha)-6(\alpha+\beta+\gamma)+27=-24$ $(\alpha-3)(\beta-3)(\gamma-3)$ $=\alpha\beta\gamma-3(\alpha\beta+\beta\gamma+\gamma\alpha)+9(\alpha+\beta+\gamma)-27$ $=39-9+81-27=84$ $\Rightarrow x^3-24x-84=0$	B1 B1 M1 A1 M1 A1 E1 **7**	Or replace x by $(x+3)$ is the equation and multiply everything out

6		Assume true for $n = k$ i.e. $\mathbf{M}^k = \begin{pmatrix} 1+4k & 8k \\ -2k & 1-4k \end{pmatrix}$ Then $\mathbf{M}^{k+1} = \begin{pmatrix} 1+4k & 8k \\ -2k & 1-4k \end{pmatrix}\begin{pmatrix} 5 & 8 \\ -2 & -3 \end{pmatrix}$ $= \begin{pmatrix} 5+20k-16k & 8+32k-24k \\ -10k-2+8k & -16k-3+12k \end{pmatrix}$ $= \begin{pmatrix} 1+4(k+1 & 8(k+1) \\ -2(k+1) & 1-4(k+1) \end{pmatrix}$ This is of the same form as $\mathbf{M}^k$ with k replaced by $(k + 1)$ Therefore if it is true for $n = k$ then it is also true for $n = k + 1$. But it is true for $n = 1$; $\mathbf{M}^1 = \begin{pmatrix} 1+4 & 8\times1 \\ -2\times1 & 1-4 \end{pmatrix}$ $= \begin{pmatrix} 5 & 8 \\ -2 & -3 \end{pmatrix}$ And so is true for any positive integer, n.	M1 M1 A1 A1 E1 M1 A1 A1 **8**	
Section B				
7	**(i)**	$\left(-\frac{1}{4},0\right)$, $(-16,0)$, $(0,-4)$	B1 (x's) B1(y) **2**	
	(ii)	$4+\frac{65x+32}{(x^2-4)} \equiv \frac{4(x^2-4)+65x+32}{(x^2-4)}$ $= \frac{4x^2+65x+16}{(x^2-4)} = \frac{(4x+1)(x+16)}{(x^2-4)}$	M1 A1 A1 **3**	
	(iii)	Asymptotes: $x = 2$, $x = -2$, $y = 4$	B1 B1 B1 **3**	
	(iv)	When $x > 2$, $(65x + 32)$, $(x - 2)$ and $(x + 2)$ are all >0 So $y > 4$	M1 A1 **2**	
	(v)		B1 B1 B1 **3**	One mark for each branch

8	**(i)**	$\|\mathbf{M}\| = 42 - 12 = 30$ Area of S = 5 sq units $\Rightarrow$ Area of T = 150 sq unit	M1 A1 **2**	
	(ii)	$\mathbf{M}^{-1} = \frac{1}{30}\begin{pmatrix} 6 & 3 \\ 4 & 7 \end{pmatrix}$	B1 B1 **2**	For $^1/_{30}$ For matrix
	(iii)	Anticlockwise rotation of 135^0 $\begin{pmatrix} \cos 135^0 & -\sin 135^0 \\ \sin 135^0 & \cos 135^0 \end{pmatrix} = \begin{pmatrix} -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}$	M1 A1 A1 **3**	For matrix with sin and cos For surd form
	(iv)	$T \to S \to U \Rightarrow \frac{1}{30}\begin{pmatrix} -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}\begin{pmatrix} 6 & 3 \\ 4 & 7 \end{pmatrix}$ $= \frac{\sqrt{2}}{30}\begin{pmatrix} -5 & -5 \\ 1 & -2 \end{pmatrix}$	M1 A1 A1 A1 **4**	Correct product For $\frac{\sqrt{2}}{30}$ or equivalent.
9	**(i)**	Sum = $\frac{4}{3}$	B1 **1**	
	(ii)	Other complex root, $\beta = 1 - \sqrt{3}\mathrm{j}$ $1 + \sqrt{3}\mathrm{j} + 1 - \sqrt{3}\mathrm{j} + \gamma = \frac{4}{3} \Rightarrow \gamma = -\frac{2}{3}$ i.e. $z = -\frac{2}{3},\ 1 + \sqrt{3}\mathrm{j},\ 1 - \sqrt{3}\mathrm{j}$	B1 M1 A1 **3**	
	(iii)	Circle, centre $1 + \sqrt{3}\mathrm{j}$, radius $\sqrt{3}$ Diagram	B1 B1 B1 B1 **4**	
	(iv)	$\frac{\alpha}{\beta} = \frac{1 + \sqrt{3}\mathrm{j}}{1 - \sqrt{3}\mathrm{j}} = \frac{1 + 2\sqrt{3}\mathrm{j} - 3}{1 + 3} = -\frac{1}{2} + \frac{\sqrt{3}}{2}\mathrm{j}$ Distance from Centre = $\sqrt{\left(1 - \frac{-1}{2}\right)^2 + \left(\sqrt{3} - \frac{\sqrt{3}}{2}\right)^2} = \sqrt{3}$.	M1 A1 M1 A1 **4**	