MEI

Mathematics in Education and Industry

MEI STRUCTURED MATHEMATICS

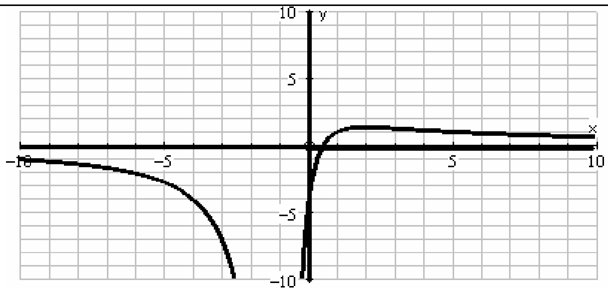
FURTHER CONCEPTS FOR ADVANCED MATHEMATICS, FP1

Practice Paper FP1-B

MARK SHEME

Qu		Answer	Mark	Comment
Section A				
1		$x^3-4x>0 \Rightarrow x\left(x^2-4\right)>0$ $\Rightarrow x>0$ and $x^2>4 \Rightarrow x>2$ and $x<0$ and $x^2<4 \Rightarrow -2<x<0$ i.e. $x>2$ and $-2<x<0$	B1 B1 **2**	
2		$x=0 \Rightarrow 2A=1 \Rightarrow A=\frac{1}{2}$ $x=1 \Rightarrow -C=3 \Rightarrow C=-3$ $x=2 \Rightarrow 2B=7 \Rightarrow B=\frac{7}{2}$ Alt: Equate coeffs $\Rightarrow \quad A+B+C=1$ $-3A-B-2C=1$ $2A=1$ $A=\frac{1}{2} \Rightarrow B+C=\frac{1}{2}, \quad B+2C=-\frac{5}{2}$ $\Rightarrow C=-3,\ B=3\frac{1}{2}$	M1 B1 for A B1 for B B1 for C **4**	
3	**(i)**	$q=-\alpha^2-\alpha\beta+\alpha\beta=-\alpha^2$	B1 **1**	
	(ii)	In the equation $p=-\left(\alpha-\alpha+\beta\right)=-\beta$ and $r=\alpha^2\beta \Rightarrow r=pq.$ $p=7, q=19, r=133$ and $7\times19=133$ $\Rightarrow q=-\alpha^2,\ \alpha^2$=-19 $\Rightarrow \alpha$ is not real. Since a cubic always has at least one root, β must be real.	M1 A1 M1 A1 A1 **5**	
4	**(i)**	$2+3j=r\left(\cos\theta+j\sin\theta\right)$ where $r=\sqrt{2^2+3^2}=3.61$ and $\cos\theta=\frac{2}{\sqrt{13}} \Rightarrow \theta=56.31$	M1 A1 A1 **3**	
	(ii)	$\left\|z-z_1\right\|=2$ is a circle, centre z_1, radius 2	B1centre B1 radius B1 correct sketch **3**	

5	**(i)**	$\mathbf{A}^{-1} = \frac{1}{2}\begin{pmatrix} 2 & 1 \\ 0 & 1 \end{pmatrix}$, $\mathbf{B}^{-1} = \frac{1}{-1}\begin{pmatrix} 0 & -1 \\ -1 & 2 \end{pmatrix}$	B1 B1 **2**	c.a.o. c.a.o.
	(ii)	$\mathbf{AB} = \begin{pmatrix} 1 & -1 \\ 0 & 2 \end{pmatrix}.\begin{pmatrix} 2 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 2 & 0 \end{pmatrix}$ $\Rightarrow (\mathbf{AB})^{-1} = -\frac{1}{2}\begin{pmatrix} 0 & -1 \\ -2 & 1 \end{pmatrix}$ $\Rightarrow \mathbf{B}^{-1}.\mathbf{A}^{-1} = -\frac{1}{2}\begin{pmatrix} 0 & -1 \\ -2 & 1 \end{pmatrix} = (\mathbf{AB})^{-1}$	M1 A1 B1 E1 **4**	
6	**(i)**	$(r+1)^2 \times r^2 - r^2 \times (r-1)^2 = r^4 + 2r^3 + r^2 - r^4 + 2r^3 - r^2$ $= 4r^3$	M1 A1 **2**	
	(ii)	$\Rightarrow 4r^3 = (r+1)^2 r^2 - r^2(r-1)^2$ $\Rightarrow 4.1^3 = 2^2.1^2 - 0$ $4.2^3 = 3^2.2^2 - 2^2.1^2$ $4.3^3 = 4^2.3^2 - 3^2.2^2$ .... $4.n^3 = (n+1)^2 n^2 - n^2(n-1)^2$ Summing both sides: $4\sum_{r=1}^{n} r^3 = (n+1)^2 n^2 - 0$ $\Rightarrow \sum_{r=1}^{n} r^3 = \frac{1}{4}(n+1)^2 n^2$	M1 A1 M1 A1 **4**	
7	**(i)**	$(1+2j)^3 - 3(1+2j)^2 + 7(1+2j) - 5$ $= 1 + 6j - 12 - 8j - 3 - 12j + 12 + 7 + 14j - 5$ $= 1 - 12 - 3 + 12 + 7 - 5 + j(6 - 8 - 12 + 14)$ $= 0$	M1 A1 A1 **3**	
	(ii)	Other complex root is the complex conjugate, $1 - 2j$	B1 **1**	
	(iii)	$k(1-2j)(1+2j) = -5 \Rightarrow 5k = -5 \Rightarrow k = -1$ So other root is -1	M1 A1 **2**	

Qu		Answer	Mark	Comment
Section B				
8	**(i)**	$x = -1$	B1 **1**	
	(ii)	When $x \to \infty,\ y = \dfrac{4(2x-1)}{(x+1)^2}$ $\to \dfrac{8x}{x^2} \to \dfrac{8}{x} \to 0$ for large x. So asymptote is x axis ($y = 0$) $\to 0$ from above as $x \to \infty$ and $\to 0$ from below as $x \to -\infty$	M1 A1 A1 M1 A1 **5**	
	(iii)	When $y = 1$, $1 = \dfrac{4(2x-1)}{(x+1)^2} \Rightarrow (x+1)^2 = 4(2x-1)$ $\Rightarrow x^2 + 2x + 1 = 8x - 4 \Rightarrow x^2 - 6x + 5 = 0$ $\Rightarrow (x-5)(x-1) = 0 \Rightarrow x = 1, 5$	M1 A1 A1 **3**	
	(iv)	 Cuts x axis when $x = ½$	B1 B1 B1 **3**	Large x General shape
9	**(i)**	$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -0.6 & 0.8 \\ 0.8 & 0.6 \end{pmatrix}\begin{pmatrix} 3 \\ 6 \end{pmatrix} = \begin{pmatrix} 3 \\ 6 \end{pmatrix}$ $\Rightarrow l$ is $y = 2x$	B1 M1 A1 A1 **4**	
	(ii)	$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 3 \\ 2 \end{pmatrix} = \begin{pmatrix} -2 \\ 3 \end{pmatrix}$ $\Rightarrow$ rotation about the origin, through $+90^o$	B1 B1 B1 **3**	Or anticlockwise
	(iii)	$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} -0.6 & 0.8 \\ 0.8 & 0.6 \end{pmatrix} = \begin{pmatrix} -0.8 & -0.6 \\ -0.6 & 0.8 \end{pmatrix}$	M1 A1 **2**	
	(iv)	$\begin{pmatrix} -0.8 & -0.6 \\ -0.6 & 0.8 \end{pmatrix}\begin{pmatrix} 1 \\ -3 \end{pmatrix} = \begin{pmatrix} -0.8+1.8 \\ -0.6-2.4 \end{pmatrix} = \begin{pmatrix} 1 \\ -3 \end{pmatrix}$ Hence invariant line is $3x + y = 0$	B1 M1 A1 **3**	

10	**(i)**	$z^2+6z+25=0 \Rightarrow z=\frac{-6\pm\sqrt{36-100}}{2}=\frac{-6\pm\sqrt{-64}}{2}$ $\Rightarrow z=-3\pm4\mathbf{j} \Rightarrow$ The roots are $-3-4\mathbf{j}$ and $-3+4\mathbf{j}$	M1 A1 A1 **3**	
	(ii)	$z_1 = -3-4\mathbf{j} \Rightarrow \lvert z_1\rvert = 5,$ $\arg z_1 = -\pi + \arctan\left(\frac{4}{3}\right) = -2.21$ radians $z_2 = -3+4\mathbf{j} \Rightarrow \lvert z_2\rvert = 5,$ $\arg z_2 = \arctan\left(-\frac{4}{3}\right) = 2.21$ radians Argand diagram	M1 A1 (both) A1 A1 B1(both) **5**	Accept 4.07
	(iii)	$\alpha+\beta=-6,\ \alpha\beta=25$ $\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta=36-50=-14$ Also $\alpha^2\beta^2=(\alpha\beta)^2=625$ $\Rightarrow x^2+14x+625=0$	M1 A1 M1 A1 **4**	