MEI

Mathematics in Education and Industry

MEI STRUCTURED MATHEMATICS

FURTHER CONCEPTS FOR ADVANCED MATHEMATICS, FP1

Practice Paper FP1-A

MARK SHEME

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Qu		Answer	Mark	Comment
Section A				
1		$x^2+3x+4=0 \Rightarrow x=\frac{-3\pm\sqrt{9-16}}{2}$ $\Rightarrow x=\frac{-3\pm \mathrm{j}\sqrt{7}}{2}$	M1 A1 A1 **3**	
2		$\frac{(3+2\mathrm{j})}{(4-\mathrm{j})}=\frac{(3+2\mathrm{j})}{(4-\mathrm{j})}.\frac{(4+\mathrm{j})}{(4+\mathrm{j})}=\frac{12+3\mathrm{j}+8\mathrm{j}-2}{16+1}$ $=\frac{10+11\mathrm{j}}{17}=\frac{10}{17}+\frac{11\mathrm{j}}{17}$	M1 A1 A1 **3**	
3	**(i)**	$\mathbf{A}^2=\begin{pmatrix}2&3\\5&-2\end{pmatrix}.\begin{pmatrix}2&3\\5&-2\end{pmatrix}$ $=\begin{pmatrix}19&0\\0&19\end{pmatrix}=19\mathbf{I}$	M1 A1 **2**	
	(ii)	$\mathbf{A}^2=19\mathbf{I}\Rightarrow \mathbf{A}^{-1}=\frac{1}{19}\mathbf{A}=\begin{pmatrix}\frac{2}{19}&\frac{3}{19}\\\frac{5}{19}&\frac{-2}{19}\end{pmatrix}$	B1 **1**	
4	**(i)**	$\mathbf{A}=\begin{pmatrix}1&2\\3&0\end{pmatrix},\ \mathbf{B}=\begin{pmatrix}2&3\\-1&1\end{pmatrix}$ $\Rightarrow \mathbf{AB}=\begin{pmatrix}0&5\\6&9\end{pmatrix},\ \mathbf{BA}=\begin{pmatrix}11&4\\2&-2\end{pmatrix}$	M1 A1 **2**	
	(ii)	$\mathbf{A}=\begin{pmatrix}1&2\\3&0\end{pmatrix},\ \mathbf{B}=\begin{pmatrix}2&3\\-1&1\end{pmatrix},\ \mathbf{C}=\begin{pmatrix}3&-2\\0&1\end{pmatrix}$ $\Rightarrow \mathbf{AB}=\begin{pmatrix}0&5\\6&9\end{pmatrix},\ (\mathbf{AB})\mathbf{C}=\begin{pmatrix}0&5\\18&-3\end{pmatrix}$ $\mathbf{BC}=\begin{pmatrix}6&-1\\-3&3\end{pmatrix},\ \mathbf{A(BC)}=\begin{pmatrix}0&5\\18&-3\end{pmatrix}$	B1 B1 B1 **3**	For AB For BC For both multiples the same
5	**(i)**	$\frac{8}{x+1}-\frac{3}{x-4}=\frac{8(x-4)-3(x+1)}{(x+1)(x-4)}$ $=\frac{5x-35}{(x+1)(x-4)}=\frac{5(x-7)}{(x+1)(x-4)}$	M1 A1 **2**	Accept $5x$ - 35

	(ii)	$\frac{5x-35}{(x+1)(x-4)} > 5 \Rightarrow (x+1)(x-4) < x-7$ $\Rightarrow x^2-3x-4 < x-7 \Rightarrow x^2-4x+3<0$ $\Rightarrow (x-3)(x-1)<0 \Rightarrow 1<x<3$ BUT only provided $(x+1)(x-4)>0$ $\Rightarrow x>4$ or $x<-1$ so never. IF $(x+1)(x-4)<0 \Rightarrow -1<x<4$ then $(x-3)(x-1)>0 \Rightarrow x>3$ or $x<1$ $\Rightarrow -1<x<1$ and $3<x<4$	M1 A1 A1 A1 A1 **5**	
6	**(i)**	$\alpha+\beta+\gamma=-p,\quad \alpha\beta+\beta\gamma+\gamma\alpha=q,\quad \alpha\beta\gamma=-r$ $(\alpha+\beta+\gamma)^2=\alpha^2+\beta^2+\gamma^2+2(\alpha\beta+\beta\gamma+\gamma\alpha)$ $\Rightarrow \alpha^2+\beta^2+\gamma^2=(\alpha+\beta+\gamma)^2-2(\alpha\beta+\beta\gamma+\gamma\alpha)=p^2-2q$	B1 M1 A1 A1 **4**	
	(ii)	$(-\alpha)+(-\beta)+(-\gamma)=-(\alpha+\beta+\gamma)=p$ $(-\alpha).(-\beta)+(-\beta).(-\gamma)+(-\gamma).(-\alpha)=(\alpha\beta+\beta\gamma+\gamma\alpha)=q$ $(-\alpha).(-\beta).(-\gamma)=-(\alpha\beta\gamma)=r$ $\Rightarrow x^3-px^2+qx-r=0$	B2,1 B1 **3**	
7		Assume true for $r = k$ i.e. $S_k = \sum_{r=1}^{k} r^2 = \frac{k(k+1)(2k+1)}{6}$ Then $S_{k+1}=\frac{k(k+1)(2k+1)}{6}+(k+1)^2$ $=\frac{1}{6}(k+1)\left(k(2k+1)+6(k+1)\right)$ $=\frac{1}{6}(k+1)\left(2k^2+k+6k+6\right)=\frac{1}{6}(k+1)\left(2k^2+7k+6\right)$ $=\frac{1}{6}(k+1)(2k+3)(k+2)$ which is the form for S_k where k+1 replaces k. So if true for k then true also for k +1 But it is true for $k = 1$; $S_1=1^2=\frac{1}{6}.1.2.3$ So it is true for S_2 and so for S_3 etc Therefore true for all k.	B1 B1 M1 A1 A1 B1 B1 B1 **8**	

Section B				
8	**(i)**	$Ax+B+\frac{C}{(x-1)}=\frac{(Ax+B)(x-1)+C}{(x-1)}\equiv\frac{x^2-x-6}{(x-1)}$ $\Rightarrow (Ax+B)(x-1)+C\equiv x^2-x-6$ Equating coeffs: $A=1,\ B-A=-1,\ C=-6\Rightarrow B=0$	M1 A1 M1 A1 A1 A1 **6**	
	(ii)	P (-2, 0), Q(3, 0) l: $x = 1$	B1 B1 B1 **3**	
	(iii)	Meet when $x=x-\frac{6}{(x-1)}\Rightarrow\frac{6}{(x-1)}=0$ which has no solution.	M1 A1 E1 **3**	
9	**(i)**	(A) (3,2) $\Rightarrow$ (2,3) (B) $\mathbf{P}^2=\begin{pmatrix}0&1\\1&0\end{pmatrix}.\begin{pmatrix}0&1\\1&0\end{pmatrix}=\begin{pmatrix}1&0\\0&1\end{pmatrix}=\mathbf{I}$ (C) Reflection in the line $y = x$	B1 **1** M1 A1 **2** B1 **1**	
	(ii)	(A) (3,2) $\Rightarrow$ (-2,3) (B) $\mathbf{Q}^2=\begin{pmatrix}0&-1\\1&0\end{pmatrix}.\begin{pmatrix}0&-1\\1&0\end{pmatrix}=\begin{pmatrix}-1&0\\0&-1\end{pmatrix}$ $\Rightarrow\mathbf{Q}^4=\begin{pmatrix}-1&0\\0&-1\end{pmatrix}.\begin{pmatrix}-1&0\\0&-1\end{pmatrix}=\begin{pmatrix}1&0\\0&1\end{pmatrix}=\mathbf{I}$ (C) Rotation through 90^0 clockwise about the origin	B1 **1** M1 A1 A1 **3** B1 **1**	
	(iii)	$\mathbf{PQ}=\begin{pmatrix}0&1\\1&0\end{pmatrix}.\begin{pmatrix}0&-1\\1&0\end{pmatrix}=\begin{pmatrix}1&0\\0&-1\end{pmatrix}$ Rotation through 90^0 clockwise about the origin followed by a reflection in the line $y = x$.	B1 B1 B1 **3**	

10	**(i)**	$\lvert z_1 \rvert = \sqrt{7}, \quad \arg z_1 = \arctan\frac{2}{\sqrt{3}} \approx 0.857$ radians	B1 B1 **2**	Accept 49.1°
	(ii)	$z_2 = 4\left(\frac{\sqrt{3}}{2} + \frac{1}{2}\mathrm{j}\right) = 2\sqrt{3} + 2\mathrm{j}$	M1 A1 **2**	
	(iii)	z_1 correctly illustrated. z_2 correctly illustrated.	B1 B1 **2**	
	(iv)	$z_2 - z_1 = 2\sqrt{3} + 2\mathrm{j} - \left(\sqrt{3} + 2\mathrm{j}\right) = \sqrt{3}$ Argand Diagram	B1 B1 **2**	Correct illustration
	(v)	The locus is a circle, centre z_1, and radius $\sqrt{3}$	M1 A1 A1 A1 **4**	Circle Centre Touching imaginary axis Through z_2