OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS 2620/1

Decision and Discrete Mathematics 1

Friday **13 JUNE 2003** Morning 1 hour 20 minutes

Additional materials:
Answer booklet
Graph paper
MEI Examination Formulae and Tables (MF12)

TIME 1 hour 20 minutes

INSTRUCTIONS TO CANDIDATES

- Write your Name, Centre Number and Candidate Number in the spaces provided on the answer booklet.
- Answer **all** questions.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The approximate allocation of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive no marks unless you show sufficient detail of the working to indicate that a correct method is being used.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The total number of marks for this paper is 60.

This question paper consists of 5 printed pages and 3 blank pages.

Section A

1 Fig.1 shows a printed circuit board with two points for external connections and three internal points. Each of the connection points is to be wired to each of the internal points.

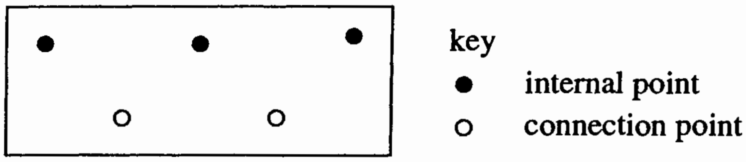


Fig. 1

(i) Show that the two connection points can each be wired directly to each of the three internal points without any wires crossing. [2]

(ii) Show that two connection points can be wired to four internal points without any wires crossing. [1]

(iii) Give the smallest numbers of connection points and internal points for which at least one crossing will be required. [2]

[Total 5]

2 Eleven boxes are to be packed into crates each of which has a weight limit of 100 kg. There are three boxes of weight 50 kg, three of weight 40 kg, three of weight 30 kg and two of weight 20 kg.

(i) Apply the first fit decreasing algorithm and state the number of crates used. [2]

(ii) Show that there is a solution using fewer crates. [2]

[Total 4]

3 Jane generates a random number by tossing four coins. She records the result from coin 1 (X_1) as 0 if it lands heads up and 1 if it lands tails up, and similarly for coins 2, 3 and 4. Jane then computes $X_1 + 2X_2 + 4X_3 + 8X_4$.

So if $X_1 = 1$, $X_2 = 0$, $X_3 = 1$ and $X_4 = 0$ then Jane's number is $1 + (2 \times 0) + (4 \times 1) + (8 \times 0) = 5$.

(i) What are the smallest and largest numbers which Jane can produce using this method? [2]

(ii) Are any of Jane's possible numbers more or less likely than others? [1]

(iii) Jane would like to extend her method so that the largest possible number is at least 99. How many coins must she toss to achieve this? [1]

(iv) Explain how Jane can generate integers between 00 and 99 inclusive, each being equally likely. [2]

[Total 6]

Section B

4 Modern Electronic Instruments Ltd. has seven depots. The distances between them in miles by direct road links are shown in Fig. 4.

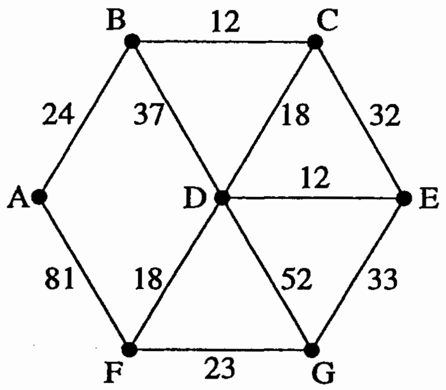


Fig. 4

(i) Communication cables are to be laid along these roads so that all depots are linked by cable, either directly or via other depots. Name an appropriate algorithm to find the minimum length of cable which is required.

Apply the algorithm, listing the order in which the roads are chosen for cabling. State the total length of cable which is needed. [4]

(ii) Use Dijkstra's algorithm to find the shortest distance and the corresponding route from depot A to depot G. [6]

An eighth depot is established at H. This new depot is connected directly by roads to A, B and E, with distances of 6, 17 and 54 miles respectively.

(iii) Starting from your solution to part (i), find the minimum length of cable needed to link the extended network. [3]

(iv) Find the shortest route from A to G in the extended network. (You are not required to apply an algorithm to show this.) [2]

[Total 15]

5 A company manufactures widgets. The manufacture of a widget involves 10 tasks as shown in Table 5.1.

Task	A	B	C	D	E	F	G	H	I	J
Immediate predecessors	–	–	–	A	A, B, C	C	C	E, F, D	E, F	G

Table 5.1

(i) Copy and complete the activity on arc network in Fig. 5.2 to represent this information. [4]

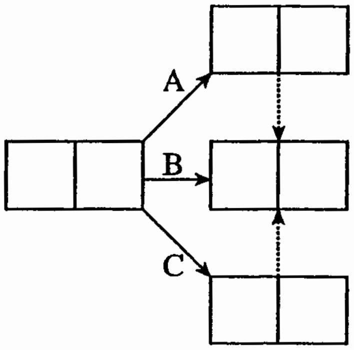


Fig. 5.2

Each task normally requires one worker, and durations in hours are given in Table 5.3.

Task	A	B	C	D	E	F	G	H	I	J
Duration (hours)	2	3	4	4	4	3	3	4	2	3

Table 5.3

(ii) Perform forward and backward passes to determine early and late event times. Give the critical activities and the minimum duration. [6]

(iii) Show that 3 workers have to be available to produce a widget in this minimum time, and show how this can be achieved. [3]

The duration of task D can be reduced by 1 hour by employing 1 extra worker on that task, or by 2 hours by employing 2 extra workers.

Similarly the duration of task E can be reduced by 1 or 2 hours by employing 1 or 2 extra workers on that task.

(iv) Give the reduced manufacturing time that is achievable, and say which duration(s) should be reduced to achieve this. [2]

[Total 15]

6 Ranil is playing a computer game. At each turn he can press X or Y. The computer then displays a window showing one of three symbols, A, B or C. Ranil's score in each case is shown in Table 6.

		A	B	C
Ranil's choice	X	15	5	0
	Y	0	10	20

Table 6

Ranil decides to press X with probability x and Y with probability $1 - x$. Whenever A shows his average score will be $15x$. Whenever B shows his average score will be $10 - 5x$. Whenever C shows his average score will be $20 - 20x$.

Ranil will choose x so that the smallest of these average scores is as large as possible. He formulates this as the following LP.

$$\begin{aligned} \text{maximise} \quad & m \\ \text{such that} \quad & m \leqslant 15x \\ & m \leqslant 10 - 5x \\ & m \leqslant 20 - 20x \\ & 0 \leqslant x \\ & x \leqslant 1 \end{aligned}$$

(i) Illustrate these inequalities on a graph, with x on the horizontal axis and m on the vertical axis. Use your graph to solve the LP. [7]

(ii) Interpret your solution. [1]

The computer displays A with probability a. It displays B with probability b and C with probability $1 - a - b$. When Ranil presses X his average score will be $15a + 5b$. When he presses Y his average score will be $20 - 20a - 10b$.

The computer is now to be programmed so that the largest of Ranil's average scores is as small as possible.

(iii) Formulate an LP to choose a and b so that the largest of Ranil's average scores is as small as possible. Your LP will need to include the inequalities $a \geqslant 0$, $b \geqslant 0$ together with a constraint on the value of $a + b$. [4]

(iv) Ranil's maximum score is minimised by choosing $a = 0.25$ and $b = 0.75$. Interpret this solution. [3]

[Total 15]

Mark Scheme

Instructions to markers

M marks are for method and are dependent on correct numerical substitution/correct application. Method marks can only be awarded if the method used would have led to the correct answer had not an arithmetic error occurred.
M marks may be awarded following evidence of an **sca** (substantially correct attempt).

M marks can be implied by correct answers.

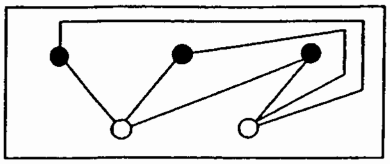
A marks are for accuracy, and are dependent upon the immediately preceding **M** mark. They cannot be awarded unless the **M** mark is awarded.

B marks are for specific results or statements, and are independent of method.

√A marks are for follow-through. This applies to **A** marks for answers which follow correctly from a previous incorrect result. Whilst mark schemes will occasionally emphasise a follow-through requirement, the default will be to apply follow-through whenever possible. The exception to this are **A** marks which are labelled **cao** (correct answer only).

MR Where a candidate misreads all or part of a question, and where the integrity/difficulty of the question is not affected, a penalty (of –1, -2 or –3) can be applied (according to the extent of the work affected), and the question marked as read.
Note that it is **not** a misread if a candidate makes an error in copying his own work.

1.

(i) 	M1 A1
(ii) Similar for $K_{2,4}$	B1
(iii) 3 and 3	B1 B1

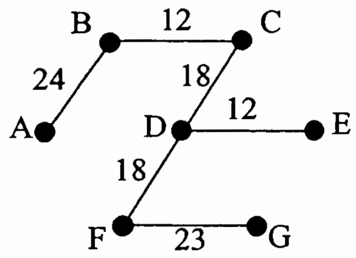
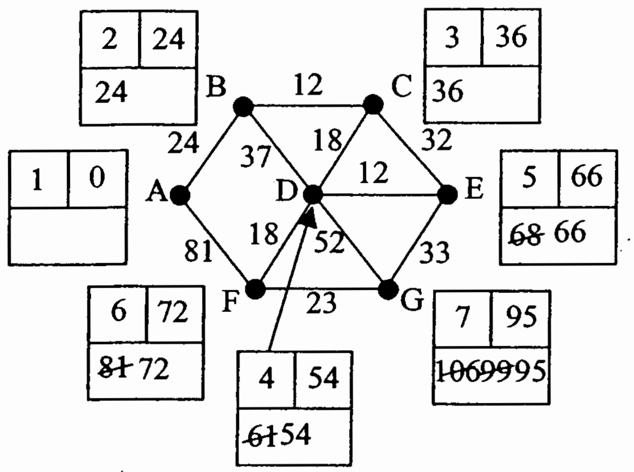
2.

(a)(i) (in tens of kilos) 5 5 / 5 4 / 4 4 2 / 3 3 3 / 2 5 crates	M1 A1
(ii) 5 5 / 5 3 2 / 4 4 2 / 4 3 3	M1 (some) complete bins A1

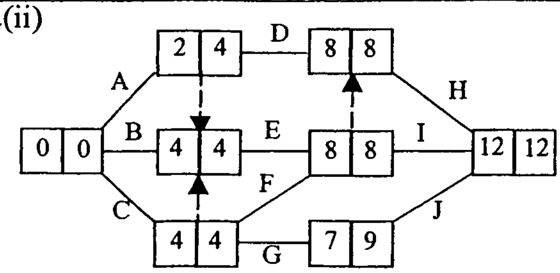
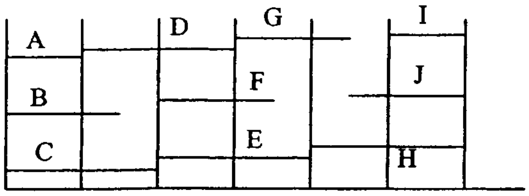
3.

(i) 0 and 15	B1 B1
(ii) No – all equally likely	B1
(iii) 7	B1
(iv) Ignore outcomes between 100 and 127 inclusive.	M1 ignore some A1

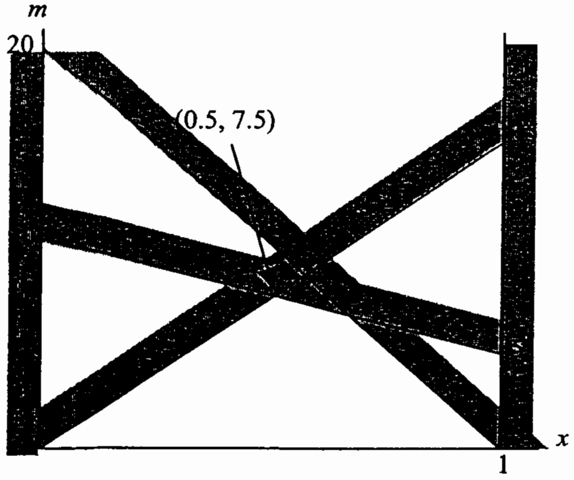
4.

(i)		M1 A1
	107 miles Prim from A: AB, BC, CD, DE, DF, FG Kruskal: (BC DE), (CD DF), FG, AB	B1 B1
(ii)		M1 Dijkstra A1 labels A2 working values (–1 each error)
	95 miles A B C D F G	B1 B1
(iii)	Add HA and HB and remove AB 106 miles	M1 A1 B1
(iv)	Now only 93 miles along A H E G	B1 B1

5.

(i)&(ii)  critical: C, E, H duration: 12 hours	M1 7 more activities A1 precedences for I A1 remaining logic A1 single end M1 A1 forward M1 A1 backward B1 B1
(iii) Σdurations/12 = 32/12 > 2	B1
e.g. 	M1 A1
(iv) Reduce E by one hour, reducing widget-making time to 11 hours.	B1 reduce E B1 11 hours

6.

(i) 	B1 B1 B1 B1 B1 M1 A1	axes labelled and scaled sloping lines shading (0.5, 7.5) (cao)
(ii) Choose x=0.5. Gives an average score of at least 7.5.	B1√	
(iii) min M st $M \geq 15a+5b$ $M \geq 20-20a-10b$ $a+b \leq 1$ $a \geq 0$ $b \geq 0$	B1 B1 B1 B1	
(iv) C will not be displayed Ranil's average score will be no more than 7.5. This is the same as his least average score in part (ii).	B1 B1 B1	

Examiner’s Report

2620 Decision and Discrete Mathematics I

General Comment

The general level of performance on this paper was disappointing. It was anticipated that the final part of the last question would be challenging, but it was also expected that candidates would find section A relatively easy. This was often not the case. It was particularly surprising to see many candidates who had not learned the basic algorithms, such as Dijkstra's.

Comments on Individual Questions

Q.1 Graphs

In part (i) candidates were expected to be able to show that $K_{2,3}$ is planar, and similarly in part (ii) that $K_{2,4}$ is planar, and in general they could. It was expected that it would then be clear that $K_{2,n}$ is planar, so that $K_{3,3}$ is the obvious answer for part (iii). Few were able to give this, though clearly little time was lost on it.

Q.2 Algorithms

Most candidates scored well on this question, although there were, inevitably, candidates who failed completely to understand the significance of "first" in "first fit decreasing". These candidates packed the first 20kg box in the fifth crate, instead of in the third crate.

Q.3 Simulation

Most candidates were able to answer parts (i), (ii) and (iii) without difficulty, which was pleasing. It was, however, surprising that very few were able to answer part (iv) correctly. The need for ignoring some sampled numbers in such circumstances has been tested on all past papers.

(i) 0 and 15
(ii) No – all equally likely
(iii) 7
(iv) Ignore outcomes between 100 and 127 inclusive.

Q.4 Networks

Not all candidates were able to make clear what algorithm they were actually using in part (i).

A significant proportion of candidates could not demonstrate an application of Dijkstra's algorithm in part (ii).

In part (iii) candidates were given some credit for just including H via HA. Similarly in part (iv) some credit was awarded for the sub-optimal improvement AHBCDFG.

(i) 107 miles
(ii) 95 miles; ABCDFG
(iii) Add HA and remove AB; 106 miles
(iv) 93 miles via AHEG

Q.5 CPA

A wide variety of wrong answers was seen in part (i). The worst used activity-on-node, even though a start was given. Others had arcs which divided or coalesced, and others had multiple joinings between pairs of nodes.

In part (iii) very few candidates computed the sum of the durations divided by the project duration to show that 3 workers are needed. Most were able to show that it can be done with 3.

(ii) Critical – C, E and H. Duration = 12 hours
(iv) Reduce E only by just 1 hour – 11 hours.

Q.6 LP

Parts (i) and (ii) were intended to be a straightforward graphical solution of a given LP. In the event many candidates made very heavy weather of it. They all seemed to know what to do, but drawing the three straight lines was often beyond them, particularly m=15x. Many otherwise successful diagrams had an inappropriate x scale, some going from x=0 to x=20.

Part (iii) was intended to challenge the better candidates to adapt the given formulation, and is covered under competence statements D1p1 and D1p4 in the "Modelling" section at the start of the syllabus (see below). Candidates would not, of course, have been expected to formulate a game theory problem as an LP without guidance. Similarly part (iv) was testing competence statement D1p3 (see below).

(ii) Choose x=0.5 – gives an average score of at least 7.5.
(iii) C will not be displayed. Ranil's average score will be no more than 7.5 – same as his least average score from part (ii)

D1p1 Be able to abstract from a real world problem to a mathematical model
D1p3 Be able to interpret and communicate results
D1p4 Be able to progressively refine a model as appropriate