OCR
RECOGNISING ACHIEVEMENT

ADVANCED SUBSIDIARY GCE

MATHEMATICS (MEI) 4771

Decision Mathematics 1

QUESTION PAPER

Candidates answer on the Printed Answer Book

OCR Supplied Materials:
- Printed Answer Book 4771
- MEI Examination Formulae and Tables (MF2)

Other Materials Required:
None

Monday 25 January 2010
Morning

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Printed Answer Book.
- **The questions are on the inserted Question Paper.**
- **Write your answer to each question in the space provided in the Printed Answer Book.** If you need more space for an answer use a 4-page answer book; label your answer clearly. Write your Centre Number and Candidate Number on the 4-page answer book and attach it securely to the Printed Answer Book.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **8** pages. Any blank pages are indicated.

Turn over

Answer all questions on the Printed Answer Book provided.

Section A (24 marks)

1 The table shows the activities involved in a project, their durations and their precedences.

Activity	Duration (mins)	Immediate predecessors
A	3	–
B	2	–
C	3	A
D	5	A, B
E	1	C

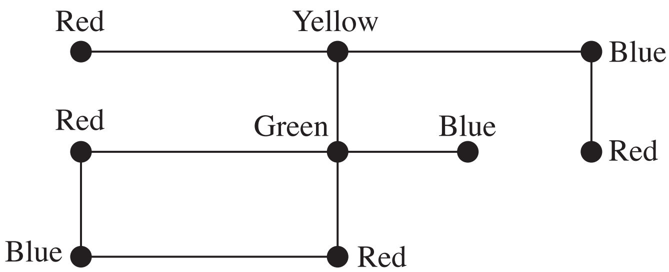
(i) Draw an activity on arc network for these activities. **[3]**

(ii) Mark on your diagram the early time and the late time for each event. Give the critical activities. **[5]**

2 The vertices of a graph are to be coloured using the following rules:

- all vertices are to be coloured
- no two vertices joined by an edge are to have the same colour.

The following graph has been coloured with four colours.



Kempe's rule allows for colours to be swapped. The rule is:

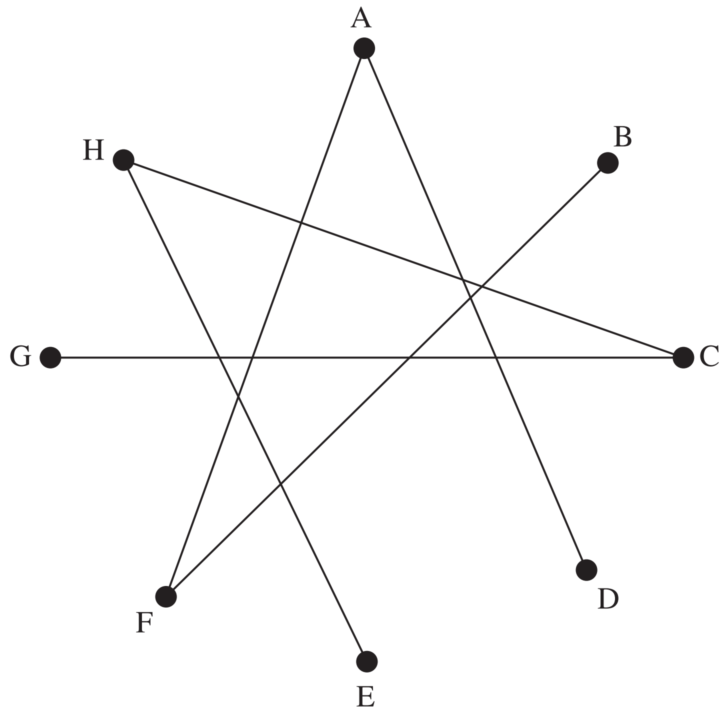
- choose two colours
- draw the subgraph consisting of the vertices coloured with these two colours, together with the edges that connect them
- in any connected part of this subgraph consisting of two or more vertices, the two colours can be swapped.

(i) Use Kempe's rule, choosing the colours blue and red.

Show that the graph can then be coloured with two colours. **[7]**

(ii) Why does Kempe's rule not constitute an algorithm for colouring graphs? **[1]**

3 Consider the following graph in which the arcs are straight lines.



(i) Explain how you know that the graph is simple. **[2]**

(ii) Explain how you know that the graph is not connected. **[2]**

(iii) On the copy of the graph in your answer book, add as many arcs as you can whilst keeping it both simple and not connected. Give the number of arcs which you have added. **[3]**

(iv) Imagine that a new graph is produced in which two vertices are connected if there is no connection between them, direct or indirect, on the original graph. How many arcs would this new graph have? **[1]**

Section B (48 marks)

4 An air charter company has the following rules for selling seats on a flight.

1. The total number of seats sold must not exceed 120.
2. There must be at least 100 seats sold, or the flight will be cancelled.
3. For every child seat sold there must be a seat sold for a supervising adult.

(i) Define two variables so that the three constraints can be formulated in terms of your variables. Formulate the three constraints in terms of your variables. **[5]**

(ii) Graph your three inequalities from part **(i)**. **[4]**

The price for a child seat is £50 and the price for an adult seat is £100.

(iii) Find the maximum income available from the flight, and mark and label the corresponding point on your graph. **[1]**

(iv) Find the minimum income available from a full plane, and mark and label the corresponding point on your graph. **[2]**

(v) Find the minimum income available from the flight, and mark and label the corresponding point on your graph. **[2]**

(vi) At £100 for an adult seat and £50 for a child seat the company would prefer to sell 100 adult seats and no child seats rather than have a full plane with 60 adults and 60 children. What would be the minimum price for a child's seat for that not to be the case, given that the adult seat price remains at £100? **[2]**

5 The matrix shows the distances in miles between towns where direct routes exist.

	A	B	C	D	E	F
A	–	22	–	12	10	–
B	22	–	–	–	–	13
C	–	–	–	6	5	11
D	12	–	6	–	–	–
E	10	–	5	–	–	26
F	–	13	11	–	26	–

(i) Draw the network. **[3]**

(ii) Use Dijkstra's algorithm to find the shortest route from A to F. Give the route and its length. **[6]**

(iii) Use Kruskal's algorithm to find a minimum connector for the network, showing your working. Draw your connector and give its total length. **[5]**

(iv) How much shorter would AD have to be if it were to be included in
(A) a shortest route from A to F,
(B) a minimum connector? **[2]**

[Question 6 is printed overleaf]

6 An apple tree has 6 apples left on it. Each day each remaining apple has a probability of $\frac{1}{3}$ of falling off the tree during the day.

(i) Give a rule for using one-digit random numbers to simulate whether or not a particular apple falls off the tree during a given day. **[3]**

(ii) Use the random digits given in your answer book to simulate how many apples fall off the tree during day 1. Give the total number of apples that fall during day 1. **[4]**

(iii) Continue your simulation from the end of day 1, which you simulated in part **(ii)**, for successive days until there are no apples left on the tree. Use the same list of random digits, continuing from where you left off in part **(ii)**.

During which day does the last apple fall from the tree? **[4]**

Now suppose that at the start of each day the gardener picks one apple from the tree and eats it.

(iv) Repeat your simulation with the gardener picking the lowest numbered apple remaining on the tree at the start of each day. Give the day during which the last apple falls or is picked.
Use the same string of random digits, a copy of which is provided for your use in this part of the question. **[4]**

(v) How could your results be made more reliable? **[1]**

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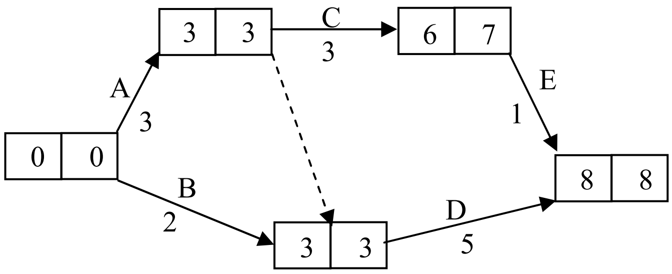
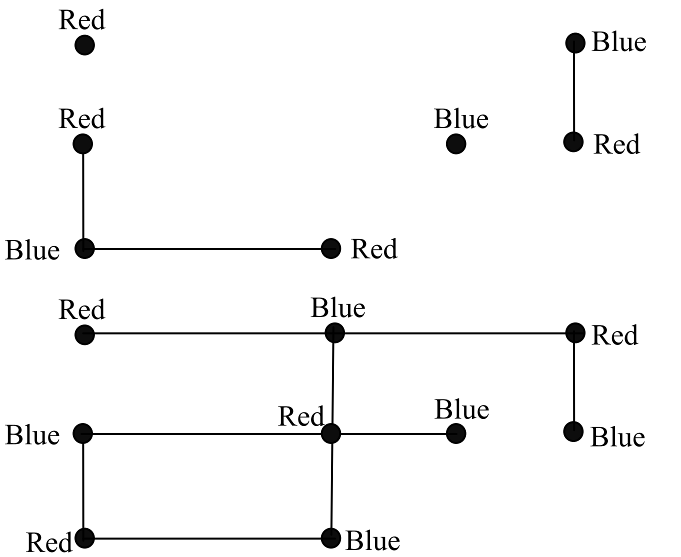
<table>
<tr><td>1</td><td>(i)
&
(ii)</td><td>

Critical activities: A and D</td><td>M1 activity-on-arc
A1 C and E OK
A1 D OK

M1 forward
A1 pass

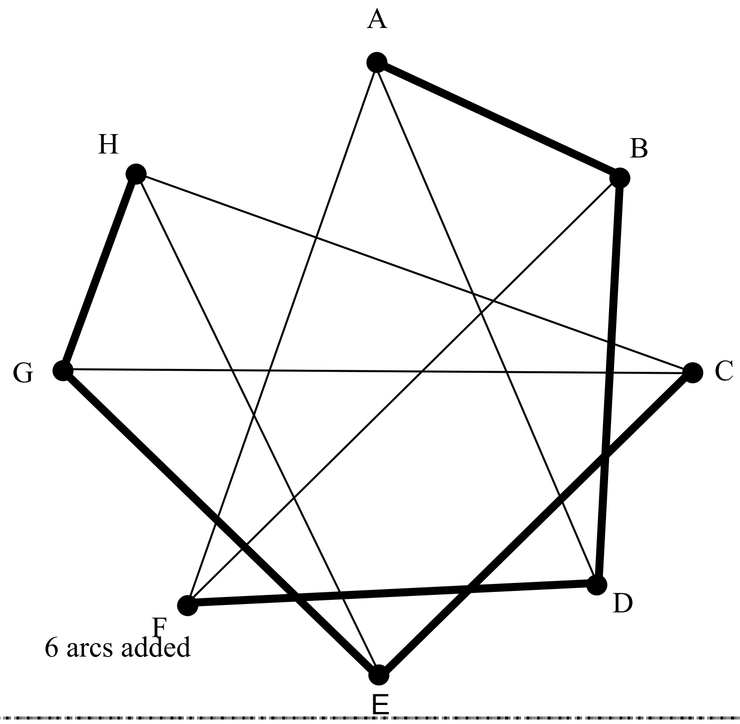
M1 backward
A1 pass

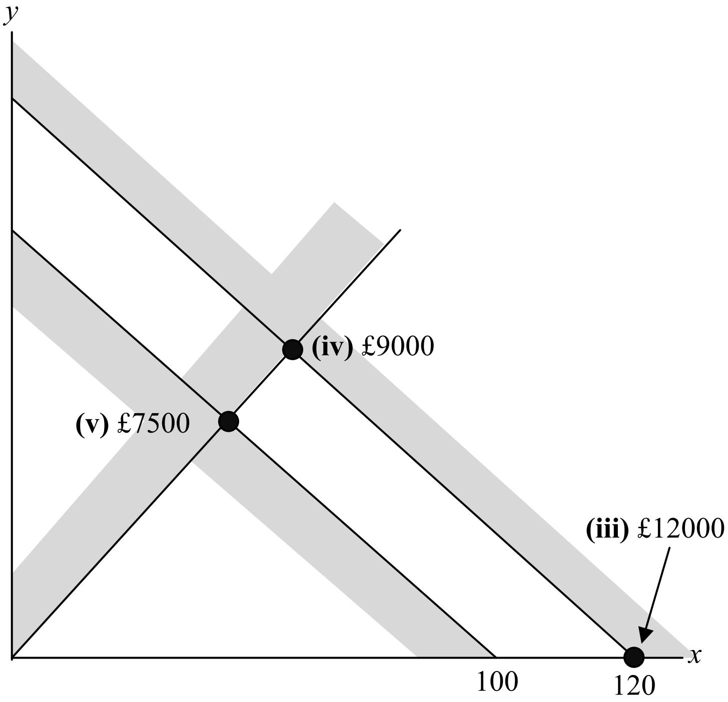
B1</td></tr>
<tr><td>2</td><td>(i)</td><td>

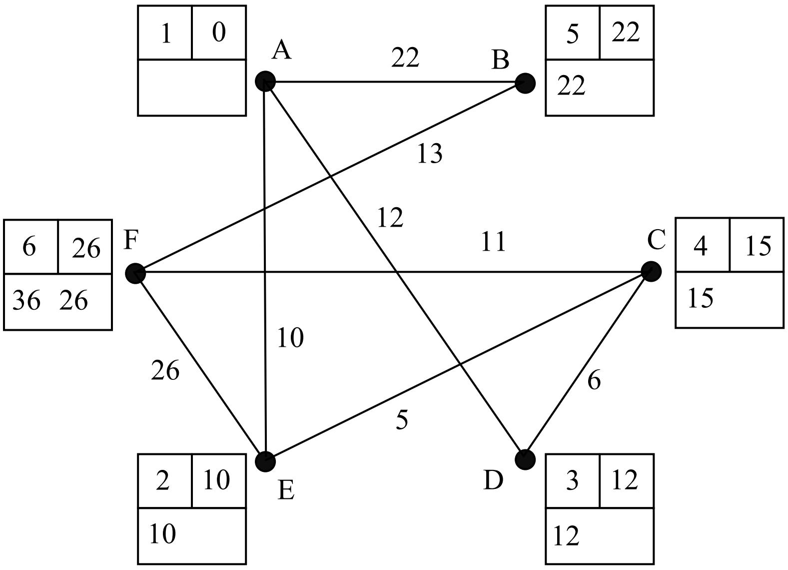
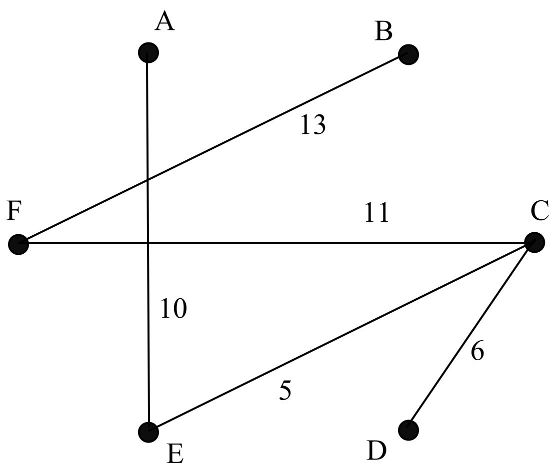
Subgraph
↓
Swap colours on connected vertices and complete</td><td>M1 subgraph
A1

M1 Changing colours
A1 top right
A1 bottom left
A1 not singletons

B1</td></tr>
<tr><td></td><td>(ii)</td><td>The rule does not specify a well-defined and terminating set of actions.</td><td>B1</td></tr>
</table>

3	**(i)**	No repeated arcs. No loops	B1 B1
	(ii)	Two disconnected sets, {A,B,D,F} and {C,E,G,H}	M1 A1
	(iii)	 6 arcs added	M1 A1 B1
	(iv)	4×4 = 16 or $\binom{8}{2} - 12 = 28 - 12 = 16$	B1

4	**(i)**	e.g. Let x be the number of adult seats sold. Let y be the number of child seats sold. $x + y \leq 120$ $x + y \geq 100$ $x \geq y$	 M1 A1 B1 B1 B1
		(ii) 	B3 lines (scale must be clear) B1 shading (axes must be clear) B1 point + amount M1 point A1 amount M1 point A1 amount
	(vi)	6000+60c > 10000 => c ≥ 67	M1 A1

5	(i) & (ii)		M1 network A1 arcs A1 lengths M1 Dijkstra A1 working values B1 order of labelling B1 labels
		shortest route: A E C F	B1
		distance: 26 miles	B1
	(iii)	CE CD AE CF ~~AD~~ BF ~~AB~~ ~~EF~~	M1 5 arc connector A1 AD not included A1 all OK, inc order
			B1
		total length of connector = 45	B1
	(iv)	A 3 miles (or length = 9) B 2 miles (or length = 10)	B1 B1

6	(i)	e.g. 0, 1, 2 → fall 3, 4, 5, 6, 7, 8 → not fall 9 → redraw	M1 ignore at least 1 A1 proportions correct A1 efficient
	(ii)	apple r n fall? 1 1 yes 2 3 no 3 8 no 4 0 yes 5 2 yes 6 7 no Three apples fall in this simulation.	M1 A2 −1 each error B1√
	(iii)	apple r n fall? 2 0 yes 3 1 yes 6 4 no apple r n fall? 6 4 no apple r n fall? 6 8 no apple r n fall? 6 0 yes 5 days before all have fallen	M1 A2 −1 each error A1√
	(iv)	apple r n fall? 1 picked 2 1 yes 3 3 no 4 8 no 5 0 yes 6 2 yes apple r n fall? 3 picked 4 7 no apple r n fall? 4 picked 3 days before none left	M1 A2 −1 each error B1√
	(v)	more simulations	B1

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General Comments

There were no significant problems with this paper. There was a wide range of outcomes. Out of 2456 entries, 20 scored full marks, and 85 scored 70 or above. At the other extreme 247 (10.4%) failed to achieve the grade E threshold of 33, and 66 scored 20 marks or fewer (... based on 2379 scripts on the system at the time of writing).

Comments on Individual Questions

1 CPA

This represented an easy starter and nearly all candidates did well on it. Those that did drop marks did so mostly in their forward pass (e.g. [2,3] for B's "j" event) or their backward pass (e.g. [3,4] for C's "i" event). A few got the logic wrong, with C and D sharing the same immediate predecessors. Only a very few attempted activity on node, which scored zero.

2 Algorithms

This was a very successful question. Some candidates were able to sail through it with full marks in very short order, while others struggled to make sense of it.

There is a philosophical issue here which emerged in one or two answers, and which again surfaces elsewhere: continuous v. discrete. It is perhaps unfortunate that "graph colouring" should be the terminology which has developed and stuck. It might have been better had it been called "graph numbering". The point is that most would regard colour as a continuous measure, corresponding to the wavelength of the light, whereas in graphical work "colour" is used as a substitute for (natural) number. Thus, in part (ii), some candidates were exercised about whether or not a computer could recognise colour, whereas a graph theorist naturally refers to colour 1, colour 2, etc.

One of the most popular answers to part (ii) was that Kempe does not provide for an initial colouring. This was allowed, although it is trivial to give an initial colouring: "Colour vertex *n* with colour *n*".

3 Graph Theory

Very few candidates were able to compose a phrase such as "no pair of vertices is directly connected by more than a single arc", even if one suspected that they understood the concept well enough. It was not uncommon for candidates to make a correct statement, and then immediately to contradict it, often within the same sentence.

Surprisingly few candidates were successful with part (iii). Zero marks were awarded if an arc was added which connected the two sets. The majority of candidates did just that.

4 **LP**

This was stunningly successfully done. Very many mostly correct answers were seen and candidates are collectively to be congratulated.

One of the more difficult marks to gain was for the use of the phrase “number of” in formulating the problem. Markers were required to be very strict with this: it is so important in understanding the underlying algebra.

5 **Networks**

This represented a fairly routine, almost context-free, trot through standard work. It caused few problems, except for part (iv). Here the correct answers were 3 miles for inclusion in the shortest route and 2 miles for inclusion in the minimum connector. The reduced distances of 9 miles and 10 miles respectively were allowed, if given instead of the reductions. However, many candidates reasoned that they needed to **force** AD into the shortest path or the minimum connector by making it shorter than its competitor arc(s). Answers of 4 and 3 for the reductions, or 8 and 9 for the reduced lengths, were extremely common, and were marked wrong.

Use of the phrase “amount of people” is to be discouraged. It suggests an underlying confusion between continuous and discrete, between measuring and counting.

6 **Simulation**

The majority of candidates managed this well enough, although not all were clear in showing what they were doing, particularly in part (iv). There it is important that the apple is picked at the start of the day, and not at the end, after falling has taken place. In most cases correct simulation could be induced from the outcomes, even when the explanation was missing, but if it could not then marks could not be awarded.

Some candidates did manage to contrive unusual apple-related simulation rules, one or two of which were OK. However, without explanation such unusual correct solutions run the risk of not being seen as correct.

Part (v) referred to “reliability”, not to “efficiency”. Only “repetition” was acceptable as an answer. Using two-digit random numbers impacts on efficiency. Considering other factors, such as the weather, relates to the simulation modelling, and not on the results of the given simulation.