MEI

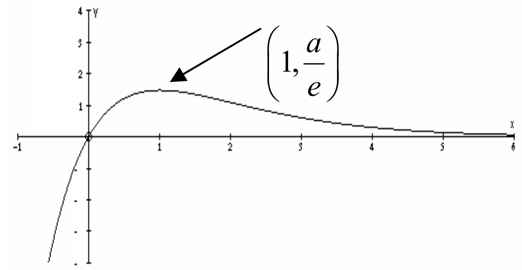
Mathematics in Education and Industry

MEI STRUCTURED MATHEMATICS

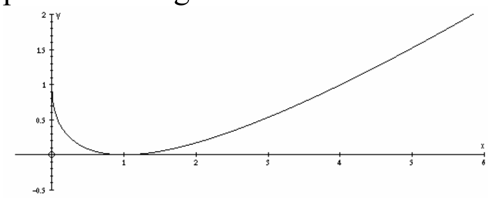
METHODS OF ADVANCED MATHEMATICS, C3

Practice Paper C3-A

MARK SCHEME

Qu		Answer	Mark	Comment
Section A				
1		Product of two numbers, one of which is even is always even. Two consecutive numbers contain an even number. *OR* acceptable alternatives	B1 B1 **2**	
2		$y=\sqrt{1+x^3}$ Let $u=1+x^3 \Rightarrow \frac{du}{dx}=3x^2$ $y=u^{\frac{1}{2}} \Rightarrow \frac{dy}{du}=\frac{1}{2}u^{-\frac{1}{2}}$ $\Rightarrow \frac{dy}{dx}=\frac{dy}{du}\times\frac{du}{dx}=\frac{1}{2}u^{-\frac{1}{2}}\times 3x^2=\frac{3}{2}\frac{x^2}{\sqrt{1+x^3}}$	M1 A1 A1 **3**	Chain rule $\frac{dy}{du}$ Answer
3	**(i)**	Substitute: $0 = a\ln 2b \Rightarrow \ln 2b = 0 \Rightarrow 2b = 1 \Rightarrow b = ½$ $1 = a\ln 2 \Rightarrow a = \frac{1}{\ln 2}$	M1 A1 A1 **3**	
	(ii)	$\lvert a\ln(bx)\rvert < 2 \Rightarrow \left\lvert\frac{\ln\frac{1}{2}x}{\ln 2}\right\rvert < 2 \Rightarrow \left\lvert\ln\frac{1}{2}x\right\rvert < 2\ln 2$ $\Rightarrow -2\ln 2 < \ln\frac{1}{2}x < 2\ln 2$ $\Rightarrow \ln\frac{1}{4} < \ln\frac{1}{2}x < \ln 4$ $\Rightarrow \frac{1}{4} < \frac{1}{2}x < 4 \Rightarrow \frac{1}{2} < x < 8$	M1 M1 A1 A1 **4**	Modulus Powers of logs
4	**(i)**	$y=axe^{-x} \Rightarrow \frac{dy}{dx}=ae^{-x}-axe^{-x}=ae^{-x}(1-x)$ $\frac{dy}{dx}=0 \Rightarrow x=1$ only at $\left(1,\frac{a}{e}\right)$ $\frac{d^2y}{dx^2}=-ae^{-x}(1-x)-ae^{-x}$: When $x=1, \frac{d^2y}{dx^2}<0$ $\Rightarrow$ Maximum	M1 A1 M1 A1 B1 **5**	Product = 0 or any equivalent argument
	(ii)		B1 B1 **2**	For curve for stationary point

5		$\int_2^3 x\mathrm{e}^{2x}\,\mathrm{d}x \qquad u = x,\ \frac{\mathrm{d}v}{\mathrm{d}x} = \mathrm{e}^{2x}$ $\frac{\mathrm{d}u}{\mathrm{d}x} = 1,\ v = \frac{1}{2}\mathrm{e}^{2x}$ $= \left[\frac{1}{2}x\,\mathrm{e}^{2x}\right]_2^3 - \frac{1}{2}\int_2^3 \mathrm{e}^{2x}\,\mathrm{d}x = \left[\frac{1}{2}x\,\mathrm{e}^{2x} - \frac{1}{4}\mathrm{e}^{2x}\right]_2^3$ $= \frac{5}{4}\mathrm{e}^6 - \frac{3}{4}\mathrm{e}^4 = 463.3$	M1 A1 M1 A1 A1 **5**	Choice of u
6		$\frac{\mathrm{d}}{\mathrm{d}x}(x\ln x) = \ln x + x\times\frac{1}{x} = \ln x + 1$ $\Rightarrow x\ln x = \int(\ln x + 1)\,\mathrm{d}x = \int \ln x\,\mathrm{d}x + x$ $\Rightarrow \int_2^3 \ln x\,\mathrm{d}x = [x\ln x - x]_2^3 = (3\ln 3 - 3) - (2\ln 2 - 2)$ $= 3\ln 3 - 2\ln 2 - 1$ $= \ln\frac{27}{4} - 1$	M1 A1 M1 A1 M1 A1 **6**	Product Integrand limits
7	**(i)**	$x = 5\sin\frac{\pi}{2} - 4\cos\frac{\pi}{2} = 5$	B1 **1**	
	(ii)	$\frac{\mathrm{d}x}{\mathrm{d}\theta} = 5\cos\theta + 4\sin\theta$: When $\theta = \frac{\pi}{2}$, $\frac{\mathrm{d}x}{\mathrm{d}\theta} = 4$ $\frac{\mathrm{d}x}{\mathrm{d}t} = \frac{\mathrm{d}x}{\mathrm{d}\theta}\times\frac{\mathrm{d}\theta}{\mathrm{d}t} = 4\times 0.1 = 0.4$	M1 A1 A1 M1 E1 **5**	4
Section B				
8	**(i)**	$\mathrm{f}(x) = \frac{x}{x^2+1} \Rightarrow \mathrm{f}'(x) = \frac{(x^2+1).1 - x.2x}{(x^2+1)^2}$ $= \frac{1-x^2}{(x^2+1)^2}$	M1 A1 E1 **3**	Formula Middle section answer
	(ii)	$\mathrm{f}'(x) = \frac{1-x^2}{(x^2+1)^2} = 0 \Rightarrow 1-x^2 = 0 \Rightarrow x = \pm 1$ When $x = 1$, $\mathrm{f}(x) = \frac{1}{1+1} = \frac{1}{2}$ i.e. $\left(1,\ \frac{1}{2}\right)$ Other stationary point is when $x = -1$, $\mathrm{f}(x) = \frac{-1}{1+1} = -\frac{1}{2}$ i.e. $\left(-1,\ -\frac{1}{2}\right)$	E1 B1 **2**	Substitute Find $\mathrm{f}(x)$
	(iii)	The graph is odd. $\mathrm{f}(-x) = \frac{-x}{(-x)^2+1} = -\frac{x}{x^2+1} = -\mathrm{f}(x)$	B1 B1 **2**	

	(iv)	$u = x^2 \Rightarrow \mathrm{d}u = 2x\mathrm{d}x$ When $x = 1, u = 1$ When $x = 4,\ u = 16$ $\Rightarrow \int_1^4 \frac{x}{x^2+1}\mathrm{d}x = \int_1^{16} \frac{1}{u+1}.\frac{1}{2}\mathrm{d}u$ $\Rightarrow a = 1,\ b = 16,\ k = \frac{1}{2}$	M1 B1 B1 A1 A1 **5**	
	(v)	Because the function is odd the area in [-1,0] is equal in magnitude but opposite in sign to the area in [0,1] So shaded area = $\int_1^4 \frac{x}{x^2+1}\mathrm{d}x + 2\int_0^1 \frac{x}{x^2+1}\mathrm{d}x$ $= \int_1^{16} \frac{1}{u+1}.\frac{1}{2}\mathrm{d}u + 2\int_0^1 \frac{1}{u+1}.\frac{1}{2}\mathrm{d}u$ $= \frac{1}{2}\left[\ln(u+1)\right]_1^{16} + 2\frac{1}{2}\left[\ln(u+1)\right]_0^1$ $= \frac{1}{2}\ln\frac{17}{2} + 2\frac{1}{2}\ln 2 = \frac{1}{2}\left(\ln\frac{17}{2} + \ln 4\right) = \frac{1}{2}\ln 34 \approx 1.763$	B1 M1 A1 M1 A1 A1 **6**	$\ln(u + 1)$
9	**(i)**	$\sqrt{y} = 1 - \sqrt{x}$ $\Rightarrow y = \left(1-\sqrt{x}\right)^2 = 1 + x - 2\sqrt{x}$ This is undefined for $x < 0$ but is defined for $x > 1$ so that part is missing. 	M1 A1 A1 B1 B1 B1 B1 **7**	Or: squaring introduces also the negative arm $x < 0$ $x > 0$ Extra range shape
	(ii)	A is $\sqrt{-x} + \sqrt{y} = 1$ for $-1 \le x \le 0$ B is $\sqrt{-x} + \sqrt{-y} = 1$ for $-1 \le x \le 0$ C is $\sqrt{x} + \sqrt{-y} = 1$ for $0 \le x \le 1$	B1 B1 B1 B1 B1 B1 **6**	
	(iii)	Area in Fig. 9.1 is $\int_0^1 \left(1 + x - 2\sqrt{x}\right)\mathrm{d}x$ $= \left[x + \frac{1}{2}x^2 - \frac{4}{3}x^{\frac{3}{2}}\right]_0^1 = 1 + \frac{1}{2} - \frac{4}{3} = \frac{1}{6}$ $\Rightarrow$ Area of shape is $\frac{1}{6} + \frac{1}{6} + 2\times\frac{1}{6} + 2\times\frac{1}{6} = 1$	M1 A1 A1 M1 A1 **5**	