MEI

Mathematics in Education and Industry

MEI STRUCTURED MATHEMATICS

METHODS FOR ADVANCED MATHEMATICS, C3

Practice Paper C3-A

Additional materials: Answer booklet/paper
Graph paper
List of formulae (MF2)

TIME 1 hour 30 minutes

INSTRUCTIONS

- Write your Name on each sheet of paper used or the front of the booklet used.
- Answer **all** the questions.
- You are permitted to use a graphical calculator in this paper.

INFORMATION

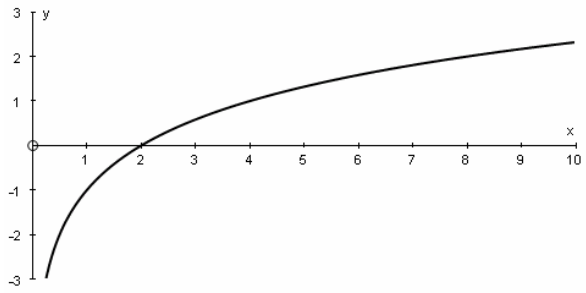
- The number of marks is given in brackets [] at the end of each question or part-question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The total number of marks for this paper is **72.**
- **You are reminded of the need for clear presentation in your answers.**

Section A (36 marks)

1 Prove that the product of consecutive integers is always even. [2]

2 Find $\frac{dy}{dx}$ when $y = \sqrt{1+x^3}$. [3]

3 The graph shows part of the function $y = a\ln(bx)$.



The graph passes through the points (2, 0) and (4, 1).

(i) Show that $b = \frac{1}{2}$ and find the exact value of a. [3]

(ii) Solve the inequality $|a\ln(bx)| < 2$. [4]

4 **(i)** Show that $y = axe^{-x}$ for $a > 0$ has only one stationary point for all values of x. Determine whether this stationary value is a maximum or minimum point. [5]

(ii) Sketch the curve. [2]

5 Find $\int_2^3 xe^{2x}\,dx$, giving your answer to 1 decimal place. [5]

6 Find $\frac{d}{dx}(x\ln x)$ and hence or otherwise find the value of $\int_2^3 \ln x\,dx$, giving your answer in the form $\ln a + b$, where a and b are to be determined. [6]

7 Two quantities, x and θ, vary with time and are related by the equation $x = 5\sin\theta - 4\cos\theta$.

(i) Find the value of x when $\theta = \frac{\pi}{2}$. [1]

(ii) When $\theta = \frac{\pi}{2}$, its rate of increase (in suitable units) is given by $\frac{d\theta}{dt} = 0.1$.

Show that at that moment $\frac{dx}{dt} = 0.4$. [5]

Section B (36 marks)

8 You are given that $f(x) = \frac{x}{x^2+1}$ for all real values of x.

(i) Show that $f'(x) = \frac{1-x^2}{(x^2+1)^2}$. [3]

(ii) Hence show that there is a stationary value at $\left(1, \frac{1}{2}\right)$ and find the coordinates of the other stationary point. [2]

(iii) The graph of the curve is shown in Fig. 8.

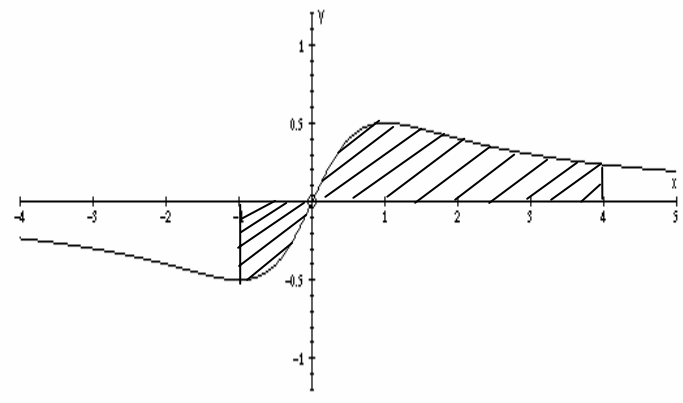


Fig. 8

State whether the curve is odd or even and prove the result algebraically. [2]

(iv) Show that $\int_1^4 \frac{x}{x^2+1}\,dx = \int_a^b k\frac{1}{u+1}\,du$, where the values of a, b and k are to be determined. [5]

(v) Hence find the area of the shaded region in Fig. 8. [6]

9 The curve in Fig. 9.1 has equation $\sqrt{x}+\sqrt{y}=1$.

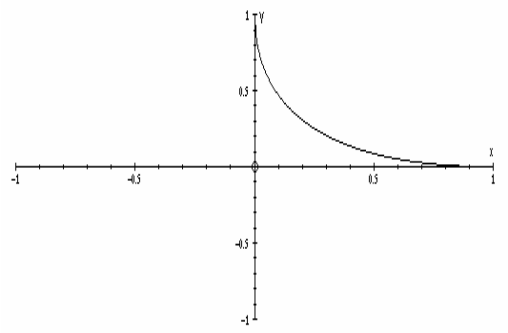


Fig. 9.1

(i) Show that this is part, but not all of the curve $y=1-2\sqrt{x}+x$.

Sketch the full curve $y=1-2\sqrt{x}+x$. [7]

(ii) Fig.9.2 shows a star shape made up of four parts, one of which is given in part (i) above.

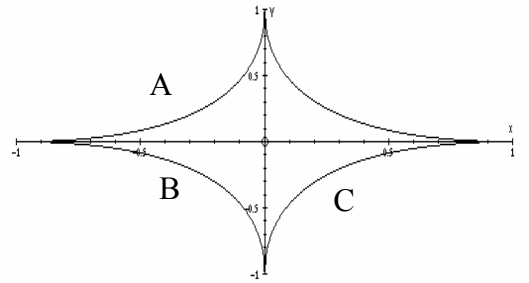


Fig. 9.2

For each of the sections of the shape labelled A, B and C, state the equation of the curve and the domain. [6]

(iii) The shape shown in Fig.9.2 is made into that in Fig. 10.3 by stretching the part of the figure for which $y > 0$ by a scale factor of 2.

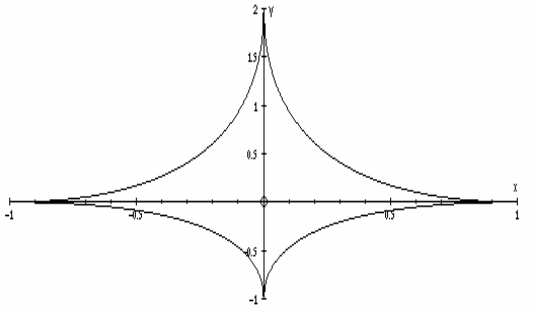


Fig. 9.3

Find the area of this shape. [5]